

THE
RETENTION OF ELEMENTARY ALGEBRA
THROUGH QUADRATICS, AFTER VARYING
INTERVALS OF TIME

BY
ANNABEL LEE WHITE

A DISSERTATION

submitted to the Board of University Studies of The Johns Hopkins University
In conformity with the requirements for the degree of
Doctor of Philosophy

1930

WASHINGTON
JUDD AND DETWEILER, INC.

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Appreciation is due to the late Dr. Edward Franklin Buchner for his unfailing kindness and interest in the problem; to Dr. Fowler D. Brooks for his untiring help and excellent suggestions; to Dr. Florence E. Bamberger for her gracious understanding; to Dr. David E. Weglein for his encouragement; to Mr. John Colbourn for his permission to use the testing material in the schools and any statistics available at the Bureau of Research; to the principals and mathematics teachers of the junior and senior high schools for their cooperation; to Dr. S. Janet Bassett for her assistance with the bibliography; and finally to the graduate students of the Educational Department of the Johns Hopkins University, who by their interest have made the graduate work in Education a real pleasure.



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CHAPTER I

THE PROBLEM AND PLAN OF ITS INVESTIGATION

A. THE PROBLEM

In this study, the problem under investigation was "The Retention of Elementary Algebra through Quadratics, after Varying Intervals of Time." Anyone who had ever taught the second course in algebra, in the public schools of Baltimore, Maryland, knew that some topics and principles of elementary algebra through quadratics, were more readily forgotten than others.

The purposes were to find out, if possible, several things, such as:

1. which topics were forgotten?
2. which topics were remembered?
3. whether such factors as chronological age, intelligence, subject preference or interest, effort, purpose, and sex played any part in the remembering of the work?

B. THE MATERIALS USED

The materials used were two tests in elementary algebra, which will hereafter be designated by Test I¹ and Test II; the Otis Self-Administering Tests of Mental Ability—Higher Examination: Form A; scores in Interest as judged by the pupils in their subject preference list; scores in effort, as judged by the teachers of plane geometry; the chronological age of all pupils concerned, which was taken on the day the Otis test was given; the purpose of the pupil, as judged by the fact that he continued or discontinued his work in algebra, after the compulsory elementary course; and the knowledge of the sex of the pupil.

Test I was made by the writer and consisted of six examples which had been analyzed into sixty-four steps. This test was to show complete mastery of the various topics of elementary algebra and hence the pupils were asked to solve each example, from start to finish. The analysis of the test, into steps, showed exactly how far each pupil was able to go with this work, and how far he was master of these topics.

Test II was made by a committee consisting of the Heads of the Mathematics Departments of the Senior High Schools

¹ See pp. 4-5 for copies of Test I and Test II.

of Baltimore, Maryland, and the supervisor of mathematics of the Junior High Schools of that city. It consisted of forty-seven examples which had been analyzed into one hundred and nine steps. In most of these examples, the pupil was asked to do one or two steps. Occasionally the example involved three or more steps, but, as a rule, the pupil was told to solve so far and no further. The complete mastery of the problem was seldom called for. This test was given so as to diagnose the pupils' difficulties. Again the analysis of the test into steps showed exactly which principle was the one that caused the error.

The scores in Tests I and II were the total number of steps correct. If in one step an error had been made, and in the succeeding steps the work was done correctly, the succeeding steps were counted correct.

For example:	$3x^2 - 4x - 7 = 0$	
These factors were wrong	$(3x - 7)(x - 1) = 0$	
These steps were counted	$3x - 7 = 0$	$x - 1 = 0$
correct	$x = 2\frac{1}{3}$	$x = 1$

The value *one* was given to each step, the more difficult steps were given the same weight as the easier ones.

The intelligence of the pupils was measured by the scores on the Otis Self-Administering Tests of Mental Ability and the chronological ages were obtained from the front pages of those tests.

The interest of the pupils was calculated on a percentile scale, in the following manner: The pupils were asked to write the subjects they were studying in the tenth grade, in their order of preference. If a pupil put mathematics first, his score was 100; second, the score was 90; third, the score was 80; and so on.

The effort scores were given to the pupils by their teachers of plane geometry. These teachers were asked to grade the effort of their pupils on a percentile scale of 100.

The purpose of the pupil was determined by the fact that he continued or discontinued his work in algebra. In Baltimore, all non-commercial pupils were required to take ninth grade algebra and tenth grade plane geometry. Eleventh grade algebra was an optional subject, for graduation from high school, but compulsory in some curricula leading on to certain colleges. No pupil was compelled to take this advanced course in algebra, unless he needed it as an entrance requirement of the college he had selected to enter.

C. THE SUBJECTS USED

The subjects used were the boys and girls of the public schools of Baltimore, Maryland. Although several hundreds of pupils took the initial test, yet when all the data had been accumulated, it was found that quite a number of pupils could not be used because they lacked a score in this or that factor. All the boys of the Polytechnic Institute had to be omitted, for in that high school the second course in algebra was given in the tenth grade, parallel to the course in plane geometry. The pupils used in this investigation were then, as follows:

Group A. One hundred thirty-nine pupils of the Eastern High School, Western High School and Baltimore City College.

Group B. Sixty-six pupils of the Forest Park High School who continued their work in eleventh grade algebra.

Group C. Seventy-six pupils of the Forest Park High School who discontinued their work in eleventh grade algebra.

Group D. Forty-five pupils of the Eastern High School.

D. TESTING PROCEDURE FOR ELEMENTARY ALGEBRA

Test I was given to all 9A¹ pupils of Baltimore, Maryland, in June 1926, at the very close of the year's work. These pupils were found in the ten² or more junior high schools and in these four senior high schools, Western, Eastern, Baltimore City College, and the Polytechnic Institute.

The commercial pupils had to be excluded from the very start, for their curriculum called for commercial arithmetic in the ninth grade, instead of algebra. After the initial test, the boys of the Polytechnic Institute had to be excluded also.

Test I was given the second time in September 1926 to the 10B³ plane geometry classes; the third time in March 1927 to the 10A plane geometry classes; and the fourth time in September 1927 to the 11B academic and technical classes, whether they were to continue or discontinue their work in algebra.

The scores of the first trial were called T_0 scores for there had been no interval for forgetting; for the second trial,

¹ A means the second semester of a grade.

² Baltimore was then almost entirely on the 6-3-3 plan, but there still were a few schools on the 8-4 plan.

³ B means the first semester of a grade.

T_3 scores, for there had been a three month interval, from June to September; for the third trial T_9 scores, for there had been a nine month interval, from June to the following March; for the fourth trial T_{15} scores, for there had been a fifteen month interval, from June 1926 to September 1927. During these intervals no algebra had been studied by the pupils used in this investigation, only plane geometry. All summer school and special pupils had been dropped out of the experiment.

Test II was given to all 9A classes, excluding commercial, in June 1927. The other three trials of the test were given to the academic and technical classes of the Forest Park High School. The second trial was given in September 1927, to the 10B plane geometry classes; the third trial in March 1928, to the 10A plane geometry classes; and the fourth trial on October 1st, 1928, to all academic and technical 11B classes, whether they were continuing or discontinuing their work in mathematics. The scores of the four trials of Test II were also called T_0 , T_5 , T_8 , and T_{16} ; the subscript represented the number of months in the interval, from the time of completion of elementary algebra to the time that the test was given. The subscripts hence told the interval during which forgetting had been taking place.

Test II was also given to forty-five 9A Eastern High School pupils in June 1927 and to the same pupils again in March 1927, after an interval of nine months. This was done so that a comparison could be made of the forgetting in an initial three month interval and an initial nine month interval.

Test I, therefore, was given to Group A and Test II to Groups B, C, and D.

By far the greater part of this dissertation is based upon the work done by Groups B and C in Test II.

E. TEST I.

Name.....Date.....
Class.....School.....
Are you taking Algebra II this year?.....

This test will not affect your standing in any way. I am endeavoring to find out what topics in Algebra I (9th grade) are remembered and those that are forgotten. The results of this investigation may help the boys and girls of the future, so please try your hardest.

Algebra I

(Use both sides of the paper)

- I. The sum of two numbers is 44. If the larger be divided by the smaller, the quotient equals $\frac{6}{5}$. Find the numbers.
- II. Solve $3x^2 - 4x - 7 = 0$ (a) by the factoring method, (b) by the completing the square method.
- III. $\begin{matrix} x - y = 3 & \text{(I)} \\ x^2 + y^2 = 29 & \text{(II)} \end{matrix}$ Solve for x and y.
- IV. (a) $\frac{\frac{2}{3} - \frac{3}{5}}{1 - \frac{1}{5}} = ?$ (b) $\frac{\frac{a}{b} - \frac{b}{a}}{1 - \frac{2}{ab}} = ?$
- V. (a) $\sqrt{8} - \sqrt{\frac{1}{2}} = ?$ (b) $2\sqrt{8} \times 3\sqrt{6} \div 6\sqrt{2} = ?$
- (c) $3 - \sqrt{x - 2} = 4$. Find x.
- VI. A man 5 feet 6 inches tall casts a shadow 2 ft. 9 in. long at the same time that a pole casts a shadow 11 ft. long. How tall is the pole?

Directions to Teachers for Giving Tests. 9th Grade Algebra.

1. Do not allow pupils to open test booklet until a signal is given.
2. Have pupils fill in blanks for name, age, class, and school.
3. Read directions on front sheet aloud to class. Make no comment.
4. Answer no questions during the test.
5. Give signal for class to turn the page and begin to work.
6. Allow exactly thirty-five minutes for the working of the test.
7. Give signal to stop and collect papers.

F. TEST II.

Score.....

Name.....Age.....

School.....Class.....

1. Read the directions carefully and do exactly as they tell you.
2. Some of the questions are not to be completely finished. Work only as far as the directions say.
3. If you cannot do a question, leave it and go on to the next.
4. Use no scrap paper. Do all of the necessary work on the front or back of these papers.
5. Try your hardest to do correctly as many questions as you can.

Algebra Test, Grade 9A

1. The ratio of angle A to angle B is 5 to 3. Find angle B when Angle A is 60 degrees.
Answer: Angle B _____ degrees.

Find the unknown terms in the following proportions:

$$2. \frac{x}{9} = \frac{3}{5}$$

$$3. \frac{x-3}{x-2} = \frac{6}{11}$$

4. If x varies as y^2 , and $x = 4$ when $y = 8$, find x when $y = 2$.
5. In order to find the value of two unknowns, we must have how many equations?

Answer.....

6. In the following pair of equations *check* to see whether these answers are correct:

$$\begin{aligned} x + y &= 12 \\ 3x - y &= 13 \end{aligned}$$

$$\begin{aligned} x &= 5 \\ y &= 7 \end{aligned}$$

Underscore correct answer
Yes No

In the equations below make the necessary changes in order to eliminate "x" by addition or subtraction. *Do not work further.*

$$7. \begin{aligned} 3x + 7y &= -5 \\ -6x + 5y &= 28 \end{aligned}$$

$$8. \begin{aligned} 9x - 4y &= 16 \\ 4x + 7y &= 51 \end{aligned}$$

Eliminate "y" in the following equations and write the resulting equations. *Do not work further.*

$$9. \begin{aligned} 3x + 5y &= -11 \\ x - 5y &= -9 \end{aligned}$$

$$10. \begin{aligned} 2x - 3y &= -9 \\ 5x - 3y &= 6 \end{aligned}$$

$$11. \frac{1}{x} + \frac{1}{y} = 7$$

$$\frac{1}{x} - \frac{1}{y} = 1$$

Using the equations given, fill in the blanks in the table below:

$$3x - 2y = 12$$

12 and 13	When $x = 0$	$y =$	
	Then $y =$	$x = -3$	

Form the equations of the following problems, but do not solve them.

14. If the numerator of a certain fraction be doubled and the denominator be increased by 5, the resulting fraction equals $7/8$. But if the numerator be decreased by 1, and the denominator be increased by 7, the resulting fraction has the value $1/3$. What is the given fraction? Let "x" be the numerator, "y" be the denominator.

Equations:

15. The perimeter of a certain rectangle is 72 inches. The width is one inch more than $2/3$ of the length. What are the dimensions? Let "x" be the number of inches in the length and "y" be the number of inches in the width.

Equations:

Write the proper answer after the equality sign:

16. $a^m \cdot a^m =$

19. $\sqrt{\frac{25a^2}{36x^4}} =$

17. $\left(-\frac{2}{3}ax^2\right)^3$

20. $\frac{8\sqrt{15}}{2\sqrt{5}} =$

18. $\frac{x^{4c}}{x^{2c}} =$

21. $\sqrt{15129} =$

Give the numerical value of:

22. $x^0 =$

23. $3^{-2} =$

24. $8^{\frac{1}{3}} =$

Express with radical signs:

25. $x^{\frac{1}{2}} =$

27. $2x^{\frac{1}{2}} =$

26. $x^{\frac{1}{4}} =$

28. $xy^{\frac{1}{3}} =$

Separate into two factors, one of which is a perfect square, the other of which contains no square.

29. $\sqrt{75} = \sqrt{\quad}$

30. $\sqrt{\frac{18}{49}} = \sqrt{\quad}$

31. $\sqrt{8a^2b^2} = \sqrt{\quad}$

Write the answer obtained by removing a factor from each radicand.

32. $\sqrt{32} = \sqrt{16 \times 2} =$

33. $\sqrt{\frac{7}{9}} = \sqrt{\frac{1}{9} \times 7} =$

34. $2\sqrt{\frac{7}{4}} = 2\sqrt{\frac{1}{4} \times 7} =$

35. Combine: $\sqrt{2ab} - 3\sqrt{2ab} - \frac{5}{2}\sqrt{2ab} =$

Write the product of:

36. $(5 - \sqrt{3})(5 + \sqrt{3}) =$

Solve by factoring:

37. $x^2 - 9x = -20$

38. $x^2 - 5x + 8 = 8$

Solve through completing the square, *but work no further.*

39. $x^2 - 4x - 7 = 0.$

40. $x^2 + 5x - 24 = 0.$

41. $2x^2 + 5x - 3 = 0.$

Make the necessary substitutions to solve for "x" or "y" *but work no further.*

42. $x^2 + y^2 = 145.$
 $x - y = 1.$

43. $xy = -91.$
 $x + y = -6.$

44. Name the curve which can be plotted from the following equation:

$$x^2 + y^2 = 25.$$

Answer:.....

Form the equations for the following problems, but do not solve them. Indicate first what the letter you use, stands for.

45. There are two consecutive integers whose product is 552. What are they?

Equation:

46. If 8 times the square of a number be increased by 2 the result is 10 times the number. What is the number?

Equation:

47. The sum of a certain number and its reciprocal is $50/7$. Is the following equation correct if "x" represents the number required?

$$x + \frac{x}{1} = \frac{50}{7}$$

Underscore the correct answer.

Yes

No

CHAPTER II

The Amount of Elementary Algebra Remembered and Forgotten during Intervals of Three, Nine, and Fifteen Months after the Completion of the Ninth Grade as Judged by the Scores on Test I and during Intervals of Five, Eight, Nine, and Sixteen Months as Judged by the Scores on Test II.

Group A which took Test I was composed of one hundred thirty-nine pupils, fifty-six of whom were boys and eighty-three girls. Seventy-two of this group continued with algebra in the eleventh grade and sixty-seven did not.

The range of scores for the first trial of the test was from 19 to 64; for the second trial, from 1 to 59; for the third trial, from 0 to 38; and for the fourth trial, from 0 to 41. As eleven pupils made a perfect score on the first trial and one pupil made a zero score on the third and two on the fourth trial, Test I did not completely measure the group. (See Table I, pp. 10).

The test, however, was reliable, for when given to a group of fifty pupils on two successive days, the coefficient of correlation was .914, for the two trials.

When the test was divided into equivalent halves, the coefficient of correlation of these halves was .852.

There was some practice effect in the taking of the test, for the mean score on the first day was 34 and for the second day 39, or a gain of 8.3%.

The fifty 9A pupils used to test the reliability and practice effect were used for that purpose alone.

The distribution of scores showed that on the first trial one hundred thirteen had scores above 40 and only twenty-six below 40; on the second trial twenty pupils had scores above 40 and sixty-six below 20; on the third trial no pupil had a score above 40 and one hundred eight had scores less than 20; and on the fourth trial one pupil had a score above 40 and one hundred five had scores less than 20.

The loss for group A, as measured by Test I, for the initial three months interval, was nearly three-fifths; for the nine-month interval, slightly over three-fourths; and for the sixteen-month interval, practically the same as for the nine-month interval. (See Table II, p. 10.)

These losses were in agreement with the findings of Ebbinghaus,¹ Radossawljewitsch,² Book,³ Bean,⁴ and others, that there was a great loss at first and then the loss decreased until the curve of forgetting became almost parallel to its axis.

TABLE I
DISTRIBUTION OF SCORES OF THE FOUR TRIALS OF TEST I. FOR GROUP A

N = 139	T ₀ *	T ₃	T ₉	T ₁₅
Scores	No. of pupils			
60 to 65	39			
50 to 60	38	11		
40 to 50	36	9	..	1
30 to 40	14	15	7	12
20 to 30	11	38	24	21
10 to 20	1	53	60	52
0 to 10		13	48	53
Mean	56.75	23.25	14.24	14.26
σ	14.38	13.15	8.49	9.63

* The subscript equals number of months between the completion of elementary algebra and the giving of the test.

Group B and C which took Test II was composed of one hundred forty-two pupils, fifty-two of whom were boys and ninety girls. Sixty-six of them continued with algebra in the eleventh grade and forty-six did not.

TABLE II
THE LOSS FOR TEST I DURING INTERVALS OF THREE, NINE, AND FIFTEEN MONTHS

June 1926 to Sept. 1926 Three months	June 1926 to March 1927 Nine months	June 1926 to Sept. 1927 Fifteen months
59.03%	76.67%	76.63%

¹ Ebbinghaus, H.: "Ueber das Gedächtnis." Untersuchungen zur experimentellen Psychologie. Leipzig, Duncker und Humboldt, 1885.

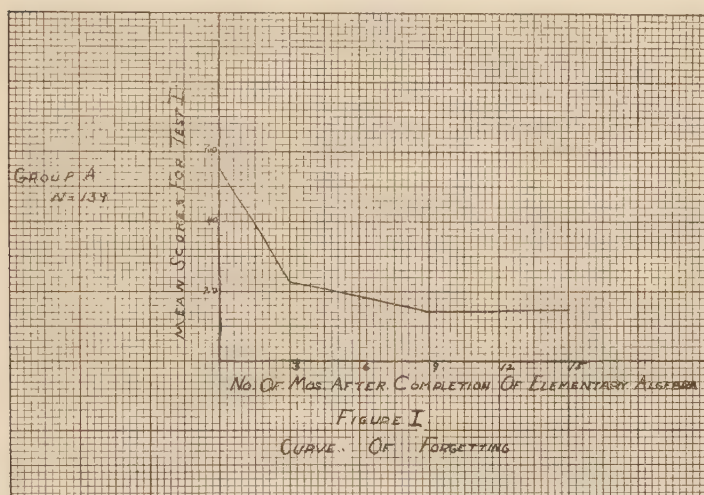
² Radossawljewitsch, P. R.: Das Fortschreiten Des Vergessens mit der Zeit. Göttingen, Druck der Dieterich'schen Univ. Buchdruckerei, 1905.

³ Book, W. F.: The Psychology of Skill; with Special Reference to its Acquisition in Typewriting. Univ. of Montana Publications of Psychology, Bulletin No. 53, Psychological Series No. 1, 1908.

⁴ Bean, C. H.: The Curve of Forgetting. Archives of Psychology, No. 2, 1912.

The range of scores for the first trial of Test II was from 35 to 105; for the second trial from 5 to 75; for the third trial, from 11 to 87; and for the fourth trial from 14 to 86. As no pupil ever made a perfect or a zero score, Test II did completely measure the group.

The test was reliable, for when given on two consecutive days, the coefficient of correlation, for the two trials, was .958. When the test was divided into equivalent halves, the coefficient of correlation of these halves was .918.



There was some practice effect shown in the taking of the test, for the mean score on the first day was 46 and on the second day 49, or a gain of 6.5%.

The fifty 9A pupils used to test the reliability and practice effect were used for that purpose alone.

The tables of the distribution of scores for group B and C showed that, on the first trial, there were thirty-six pupils above 80 and only three below 40; on the second trial, there was not one pupil above 80 and seventy-three below 40; on the third trial, there was one pupil above 80 and seventy-five below 40; and on the fourth trial, there was one pupil above 80 and seventy-three below 40.

Test II was also taken by group D, which was composed of forty-five girls. The interval between the first and second trials of the test, for these pupils, was nine months.

The comparison of the distribution tables of group D with those of group B and C revealed the fact that the loss after nine months was much greater than that after five months. On the initial trial of Test II, 31.1% of group D were above 80 as compared with 25.4% of group B and C; and 2.5% of group D were below 40 as compared with 2.1% of group B and C. After an interval of nine months, group D had no pupil above 60, while after an interval of five months, group B and C had 7.7%; and

TABLE III

DISTRIBUTION OF SCORES OF THE FOUR TRIALS OF
TEST II FOR GROUP B AND C

N = 142	T ₀	T ₅	T ₈	T ₁₆
Scores	Number of pupils			
100-109	2			
90-100	15		1	1
80-90	19			
70-80	34	4	4	5
60-70	34	5	9	20
50-60	25	24	19	21
40-50	10	36	34	22
30-40	3	38	33	34
20-30		25	33	22
10-20		9	9	17
0-10		1		
Mean—	58.17	39.06	39.83	40.13
σ	15.22	13.9	14.27	15.29

* The subscript equals the number of months between the completion of elementary algebra and the giving of the test.

73.3% of group D were below 40 as compared with 51.4% of group B and C. The nine month interval caused a much greater forgetting than the five month interval.

The loss of Group B and C for the first five months was not quite one-third and thereafter there was no loss but a slight gain. (See Table V.)

When the loss of group D was compared with the loss of group B and C, it was found that an initial interval of nine months gave a loss of 55.9%, which was 23.1% more than the loss for an initial interval of five months. After the interval of five months, between the first two trials of the

TABLE IV

DISTRIBUTION OF THE SCORES OF THE TWO TRIALS OF TEST II
FOR GROUP D

N = 45	*T ₀	T ₉	T ₀	T ₉
Scores	No. of pupils	Scores	No. of pupils	
100-109		40-50	2	9
90-100	5	30-40	1	11
80- 90	9	20-30		17
70- 80	13	10-20		5
60- 70	10	0-10		
50- 60	5			
		Mean =	72.11	31.78
		σ =	14.39	11.18

* The subscript equals the number of months between the completion of elementary algebra and the giving of the test.

test, group B and C lost no more, for after eight months their loss was 31.5% and after sixteen months 30.01% The initial five month interval was better for remembering than an initial nine month interval.

TABLE V

THE LOSS FOR TEST II FOR INTERVALS OF FIVE, EIGHT, AND SIXTEEN
MONTHS

June 1927 to Nov. 1927	June 1927 to Feb. 1928	June 1927 to Oct. 1928
Five months	Eight months	Sixteen months
32.8%	31.5%	30.01%

CHAPTER III

ANALYSIS OF DIFFICULTIES AND KINDS OF ERRORS

When an analysis of the difficulties and kinds of errors found in this investigation was being made, this fact had to be taken into consideration, that two types of tests were used. Test I was intended to test complete mastery of the principles involved and hence the pupils were asked to solve each question fully; while Test II was formulated so as to diagnose individual difficulties and hence the pupils were asked to work a step or two of each problem, and in a few cases were asked to complete the solution.

A. ANALYSIS OF DIFFICULTIES AND KINDS OF ERRORS REVEALED BY THE EXAMPLES OF TEST I

When the four trials of Test I* were marked on the scale of the sixty-four steps, into which Test I had been analyzed, the same mistakes were found to have been made by a number of individuals, and frequently had been made more than once by the same individual. Some of the mistakes that were noted were the following:

1. Numerical errors were responsible for approximately 10% of the mistakes. This carelessness did not increase with the length of the time interval but remained consistently about one mistake out of every ten. (See Table VI, p. 16.)

2. The failure to change the sign of a term, when transposing it from one side of an equation to the other, was an error that was made twenty-one times on the first trial, twenty-two on the second, sixteen on the third, and nine on the fourth. The per cents of error on the four trials of the test were 2.2, 6.2, 7.1, and 4.1 respectively. This step occurred eleven times during the test. (See Table VI, pp. 16, 17, 18, 21, I-4,9; IIa-3, 5; IIb-7, 8; III-8, 9, 12, 13; V-c 3.)

3. Clearing an equation of fractions occurred twice, during the test. (See Table VI, pp. 16, 22, I-3; VI-4.) This was done incorrectly, eleven times on the first trial, thirty-one times on the second, twenty-three times on the third, and thirty-three times on the fourth. The per cents of error on the four trials of the test were 4.9, 23.3, 15.7, and 31.4 respectively.

* The one hundred thirty-nine pupils of Group A took Test I.

4. The solution of quadratic equations of one unknown (see Table VI, p. 17, II a, b) suffered losses of approximately 50% and 80%, respectively, in the sixteen month interval. The completing the square method lost 30% more than the factoring method. This was not surprising to the writer, as she knew that absolutely mechanical processes of many consecutive steps had few associative bonds to keep them in memory.

5. The solution of two simultaneous equations (see Table VI, p. 18, III) when one was linear and the other quadratic, dropped off badly—again mechanical work with few associative bonds. Of the one hundred seven who started out with all thirteen steps correct, only two kept this correctness consistently, until the end.

6. Example IVa was the handling of numerical fractions, placed in the test as a teaching device for example IV-b, which was the handling of algebraic fractions. These two examples lost the same amount, for seventy-one pupils got both examples entirely correct the first time and twenty-one the fourth time. (See Table VI, p. 19, IV a and b.)

7. The three simple questions in radicals lost heavily. (See Table VI, pp. 20, 21, V a, b, c.) In example V-a, one hundred nineteen pupils had the first step of separating the radicand of $\sqrt{8}$ into $\sqrt{4 \times 2}$ correct the first time, and then the number of pupils with correct solutions dropped to sixty, twenty-seven, and sixteen, with eight getting it consistently right. The separation of $\sqrt{1/2}$ into $\sqrt{1/4 \times 2}$ lost even more heavily. One hundred one had it correct the first time, thirty-eight the second, twelve the third, and ten the fourth. Even those pupils who simplified the two surds into $2\sqrt{2} - 1/2\sqrt{2}$ were not able to finish the example correctly.

Example V-b which involved multiplication and division of surds had fewer correct solutions than Example V-a. In the beginning, only sixty-nine out of one hundred thirty-nine had solved it completely and this number dropped to twenty-two, three, and two.

The solution of an irrational equation, example V-c, started out with one hundred pupils getting the first three steps correct. In the beginning, only seventy-one of these realized that the answer must be checked, to see if it were a root of the original equation. After three months 29% of those one hundred pupils remembered how to solve this example and 14% remembered the necessity of checking it.

TABLE VI

THE ANALYSIS OF TEST I INTO SIXTY-FOUR STEPS, WITH THE
NUMBER OF PUPILS GETTING EACH STEP CORRECT,
ON THE FOUR TRIALS OF THE TEST.

Group A—N=139 Example I	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₅	All 4 times
$x+y=44$ (1) . . .	Translation from English into al- gebra	1	137	121	109	94	67
$\frac{x}{y} = \frac{6}{5}$ (2)		2	133	122	109	94	75
$5x=6y$ (2)	Clearing of frac- tions	3	125	94	89	68	47
$5x-6y=0$ (2)	Transposing	4	118	84	82	61	44
$5x+5y=220$ (1)	Multiplying the first equation by 5	5	119	76	60	40	40
$-11y = -220$	Subtracting equation (1) from equation (2)	6	118	74	69	54	36
$y=20$	Dividing the re- sult by -11 .	7	118	77	79	55	35
$x-20=44$	Substituting the value of y in equation (1)	8	114	77	80	53	33
$x=24$	Transposing	9	113	75	76	53	32
All nine steps correct			111	72	71	53	28

From a study of the above analysis, it seems that clearing an equation of fractions must have been the most difficult step, as the loss was the greatest between the second and third steps.

There were two other solutions of example 1 that were used by a few pupils. The first step, in both cases, was weighted five, for the total score.

$$\begin{array}{rcl}
 \frac{x}{44-x} & = & \frac{6}{5} \\
 5x & = & 264-6x \\
 11x & = & 264 \\
 x & = & 24 \\
 44-x & = & 20
 \end{array}$$

$$\begin{array}{rcl}
 6x+5x & = & 44 \\
 11x & = & 44 \\
 x & = & 4 \\
 6x & = & 24 \\
 5x & = & 20
 \end{array}$$

Example II -a. $3x^2-4x-7=0$	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₅	All 4 times
$(3x-7)(x+1)=0$ $3x-7=0$ $x+1=0$ $3x=7$ $x=2\frac{1}{3}$ $x=-1$	Factoring	1	129	93	39	66	29
	Equating each factor to zero	2	129	91	46	64	29
	Transposing	3	129	84	42	62	29
	Dividing by 3	4	121	90	43	61	28
	Transposing	5	120	84	42	58	27
All 5 steps correct			118	84	35	58	24

From a study of this summary it was concluded that at least 90% of those pupils who got started on this example finished it correctly.

Example IIb. $3x^2-4x-7=0$	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₅	All 4 times
$3x^2-4x=7$ $x^2-\frac{4}{3}x=\frac{7}{3}$ $x^2-\frac{4}{3}x+\frac{4}{9}=\frac{7}{3}+\frac{4}{9}$ $x-\frac{2}{3}=\pm\frac{5}{3}$ $x-\frac{2}{3}=\frac{5}{3}$ $x-\frac{2}{3}=-\frac{5}{3}$ $x=\frac{7}{3}$ $x=-1$	Transposing the constant term	1	121	56	24	37	6
	Making the coefficient of x^2 equal to 1	2	121	49	22	37	1
	Completing the square	3	114	35	14	34	1
	Extracting the square root	4	113	26	4	25	1
	Forming the two equations, due to the double sign	5	113	27	2	21	1
		6	113	25	2	21	1
	Transposing	7	109	26	2	24	2
		8	102	25	2	22	1
All 8 steps correct			93	25	2	22	1

From a study of these figures, the conclusion was drawn that those pupils who got the first step correct, also got the second step; and that the third and fourth steps were

the two most difficult for the pupils. From years of experience in teaching mathematics, the writer knew that the steps which involved the completing of the square and the extracting of the square root would be the place where the forgetting would begin. Close to one hundred per cent of those pupils who weathered the first four steps, were able to finish the example correctly.

The slight increase in the scores, on the fourth trial of the test, was due to the pupils who were continuing their work in algebra in the eleventh grade.

Example III. $x - y = 3$; $x^2 + y^2 = 29$	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₅	All 4 times
$x = 3 + y$	Finding the value of one unknown in the linear equation	1	135	69	29	20	10
$9 + 6y + y^2 + y^2 = 29$	Substituting this value in the quadratic equation	2	135	62	18	18	5
$2y^2 + 6y - 20 = 0$	Combining like terms and transposing	3	132	48	16	12	3
$y^2 + 3y - 10 = 0$	Dividing by 2	4	130	42	12	12	3
$(y + 5)(y - 2) = 0$	Factoring	5	128	31	9	8	3
$y + 5 = 0$	Equating each factor to zero	6	126	31	10	8	3
$y - 2 = 0$		7	123	31	10	8	3
$y = -5$		8	123	29	9	8	3
$y = 2$	Transposing	9	122	29	8	8	3
$x + 5 = 3$		10	116	29	8	8	3
$x - 2 = 3$	Substituting	11	116	26	8	8	2
$x = -2$		12	116	24	4	8	2
$x = 5$	Transposing	13	107	22	4	8	2
All 13 steps correct			107	22	4	8	2

From a study of these tabulations, the conclusion was drawn that almost one half of the pupils had forgotten the initial step, after an interval of three months; almost four-fifths of the group after nine months, and almost six-sevenths of the group after fifteen months.

Example IV-a. $\frac{2}{3} + \frac{3}{5} =$ $1 - \frac{1}{5} =$	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₅	All 4 times
$\frac{10+9}{5} =$ $\frac{5-1}{5}$	Changing the fractions in the numerator to fifths	1	124	79	78	63	36
	Changing the whole no. in the denominator to fifths	2	124	79	77	63	38
$\frac{19}{5} \times \frac{5}{4} =$	Inverting the divisor	3	121	66	60	51	27
$\frac{19}{5} \times \frac{5}{4} =$	Cancellation	4	113	60	60	51	27
$\frac{19}{4}$ or $4\frac{3}{4}$	Getting the answer	5	71	40	32	24	9
All 5 steps correct			71	40	32	21	9

The loss in this example was most surprising to the writer.

Example IV.-b. $\frac{a}{b} - \frac{b}{a} =$ $1 - \frac{2}{ab}$	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₅	All 4 times
$\frac{a^2-b^2}{ab}$	Subtraction of fractions in the numerator	1	91	48	37	43	12
$\frac{ab-2}{ab}$	Changing the one to $\frac{ab}{ab}$	2	80	38	33	29	8
$\frac{a^2-b^2}{ab} \times \frac{ab}{ab-2}$	Inverting the divisor	3	90	40	23	27	7
$\frac{a^2-b^2}{ab} \times \frac{ab}{ab-2}$	Cancelling the common factor	4	71	29	21	23	5
*All 4 steps correct			71	29	21	21	5

*The getting the answer was not counted a step.

The writer found from a scrutiny of the score sheets for each individual, that 98%, or practically the whole group, who had the algebraic fractions correct, had the arithmetical fractions also correct. This seemed to show a carry over from IV-a to IV-b.

Example	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₆	All 4 times
V.-a. $\sqrt{8} - \sqrt{\frac{1}{2}}$							
$\sqrt{4 \times 2}$	Separating the radicand into two factors, one of which contains all the perfect squares	1	119	60	27	16	8
$\sqrt{\frac{1}{4} \times 2}$		2	101	38	12	10	4
$2\sqrt{2}$	Taking the square root of the perfect square factors and placing the result as the coefficient of the radical	3	88	28	4	6	1
$\frac{1}{2}\sqrt{2}$		4	86	26	4	5	1
$1\frac{1}{2}\sqrt{2}$	Subtracting these coefficients	5	78	23	3	5	1
All 5 steps correct			78	23	3	2	1

The simplification of the fractional radicand was more difficult for the pupils than the simplification of the integral radicand, in every trial of the test, as seen from the results of steps 1 and 2.

Example	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₆	All 4 times
V.-b. $2\sqrt{8} \times 3\sqrt{6} \div 6\sqrt{2}$							
$= 6\sqrt{48} \div 6\sqrt{2}$	Multiplying coefficients and radicands	1	89	35	10	8	4
$= \sqrt{24}$	Dividing coefficients and radicands	2	89	32	7	6	2
$= \sqrt{4 \times 6}$	Separating the radicand into 2 factors one of which contains all the perfect squares	3	89	31	6	6	1
$= 2\sqrt{6}$	Extracting the square root of the perfect square	4	72	22	3	2	1
All 4 steps correct			69	22	3	2	1

At the close of the year's work in elementary algebra, only 64% of the pupils knew how to multiply and divide surds; after three months, less than 30% could still do the work; and after nine and sixteen months, less than 10%. This surely was a great loss.

Example	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₆	All 4 times
V.-c. $3 + \sqrt{x-2} = 4$							
$\sqrt{x-2} = 1$	Combining the rational terms	1	100	29	5	4	1
$x-2 = 1$	Squaring both sides of the equation	2	100	29	5	4	1
$x = 1 + 2$	Transposing	3	100	29	5	5	1
$x = 3$	Combining	4	71	14	2	2	0
Proof: $3 + \sqrt{3-2} = 4$	Substituting the root for the unknown	5	71	14	2	2	0
$3 + 1 = 4$	Taking the square root	6	70	12	2	1	0
All 6 steps correct			68	11	1	0	0

Here, again, the loss was very great. This work with radicals came at the end of the year and evidently not sufficient overlearning had taken place. After marking the parts of Example V, the writer felt like agreeing with Worcester¹, that it might be advisable "to leave off the last car."

Example VI. A man 5 ft., 6 in. tall casts a shadow 2 ft., 9 in. long, at the same time that a pole casts a shadow 11 ft. long. How tall is the pole?	Analysis	No. of Pupils Getting the Indicated Step Correct					
		Steps	T ₀	T ₃	T ₉	T ₁₆	All 4 times
$\frac{5\frac{1}{2}}{x} = \frac{2\frac{3}{4}}{11}$	Translating from English into algebra	1	105	56	43	50	19
$\frac{11}{x} = \frac{11}{11}$	Changing mixed numbers to improper fractions	2	88	39	38	35	15
$\frac{2}{x} = \frac{4}{11}$	Product of means = product of extremes	3	88	39	38	30	15
$\frac{11}{4}x = \frac{121}{2}$	Clearing of fractions	4	85	36	35	25	13
$11x = 242$	Dividing by 11	5	73	36	32	28	7
x = 22			73	32	30	25	7
All 5 steps correct							

A few pupils worked this example by drawing to scale. The total score for this correct solution was counted five.

The first and last examples were the translation of English sentences into algebra. The scores for these translations were better than nearly all the scores in the test.

B. ANALYSIS OF DIFFICULTIES AND KINDS OF ERRORS REVEALED BY THE EXAMPLES OF TEST II.

1. When the four trials of Test II² were marked on the scale of the one hundred nine steps into which Test II had been analyzed, the following mistakes were noted.

1. Although the translation of an English sentence into an algebraic statement was well done, yet there were some

¹ Worcester, Dean A.: The Permanence of Learning in High School Subjects. Journal of Educational Psychology, May 1928.

² The one hundred forty-two pupils of groups B and C and the forty-five pupils of group D took Test II.

mistakes that were made several times by a number of pupils. (See Table VII, pp. 26-35, I-1, 2; IV-1, 3; XIV, XV, XLV; XLVI; XLVII.)

In example I, the scores improved and in the other examples, at least 70% of the pupils who had the step correct the first time, also had it correct the fourth time.

Some of the mistakes that were found in these translations were the following:

In example XIV—step 1—if the numerator of a fraction be doubled— X^2 was used by approximately 35% of the pupils.

In example XV—step 1—the perimeter of a rectangle was given as $x+y$ or even as xy ; only forty out of one hundred forty-two gave it as $2x+2y$, on the first trial.

In example XV—steps 2 and 3—the width is one inch more than $\frac{2}{3}$ of the length, the pupils were uncertain which dimension was to have $\frac{2}{3}$ of it taken and whether the one was to be added or subtracted, and from which quantity this was to be done. About 25% of the pupils had this step correct.

In example XLV—step 1—two consecutive numbers were given as x and $2x$; sixty-six pupils out of one hundred forty-two gave x and $x+1$ correctly the first time.

In example XLVII, the meaning of the word reciprocal was known originally by fifty-six out of one hundred forty-two pupils.

2. In clearing an equation of fractions (see Table VII, pp. 26, 27, I-3; II-1; III-1) about 75% of the pupils seemed to be able to apply the axiom, equals multiplied by equals give equals; although, as most of the fractional equation had only four terms, they might have been using the principle, that in a proportion, the product of the means equals the product of the extremes.

3. In transposing terms from one side of an equation to the other, only seventy-nine out of one hundred forty-two got this step correct the first time. (See Table VII, p. 27, III-2).

The cases of transposition that were dependent upon the knowledge of the first step, in the solution of a quadratic equation of one unknown, lost heavily, about 66%. The loss here was due to the forgetting of the method. (See Table VII, pp. 34, 35, XXXVII-1; XXXVIII-1; XXXIX-1; XLI-1; XLI-1.) As in Test I, more overlearning was needed to keep these mechanical processes, so that they could be used with certainty. (See Table VI, pp. 16.)

4. In the solution of quadratic equation of one unknown, by the factoring method, almost 100% of those pupils who equated the equation to zero, correctly, got the right factors. However, there was a falling off of about 50% in the number of pupils who were able to do these steps the next time. (See Table VII, p. 34, XXXVII-1, 2.)

Example XXXVIII presented a real difficulty to the pupils. $x^2 - 5x - 8 = 8$ (see Table VII, p. 34)

step 1 $x^2 - 5x = 0$

After step 1, less than 10% of the pupils knew what to do; evidently they were more used to factoring a quadratic trinomial.

5. In the solution of quadratic equation in one unknown, by the completing the square method, the loss was about 30% greater than by the factoring method. (See Table VII, pp. 34, 35, XXXIX, XL, XLI.) The same result was found in Test I. (See Table VI, p. 17, II a, b.)

6. The fact that two equations were needed to solve an example containing two variables was known by practically 100% of the group. (See Table VII, p. 20, V.)

7. The fact that the graph of the equation $x^2 + y^2 = 25$ was a circle, was known by less than one-third of group. (See Table VII, p. 35, XLIV.)

8. The ability to decide whether values of x and y were correct for a pair of equations was done perfectly by about 97% of the group, all four times. (See Table VII, p. 27, VI.)

9. The ability to substitute a value of one unknown, and to find a corresponding value, for the other unknown, was remembered by about 90% of the pupils who had the examples correct the first time. (See Table VII, p. 29, XII and XIII.)

10. The handling of linear equations, in two unknowns, by the method of elimination by addition or subtraction, was known by 75% of the pupils and remembered very well by them. (See Table VII, pp. 28, 29, IX, X, XI.)

11. The ability to fix linear equations of two unknowns, so that one unknown could be eliminated by addition or subtraction, was known by 90% of the pupils and remembered by most of them. (See Table VII, pp. 27, 28, VII, VIII.)

12. The $\pm$ sign had absolutely no significance for the vast majority of these pupils. It was known originally by less than 10% of the pupils and forgotten by practically every one of them. (See Table VII, pp. 30, 33, XIX-1; XXXII-1; and XXXIII-1.)

13. The facts that exponents were added in multiplication and subtracted in division, when the quantities involved had the same bases, was known by about 75% of the pupils in the first trial and remembered by at least 50% of the whole group. This was true even when the exponents were literal. (See Table VII, pp. 29, 30, XVI-1; XVIII-4, 5; XIX-4, 5.)

14. The square root of a number, which had to be worked out, by the mechanical process of undoing $a^2 + 2ab + b^2$ was known, in the beginning, by less than one-fourth of the pupils and not always remembered by them. (See Table VII, p. 31, XXI.)

15. The meaning of a fractional exponent was known, in the beginning, by 40% more pupils, when used with literal bases than with numerical bases. (See Table VII, pp. 31, 32, XXIV-1; XXV-1; XXVI-1; XXVII-2; XXVIII-1.)

16. The meaning of the zeroth exponent was known by 94% of the group and remembered by about 70% of them. (See Table VII, p. 31, XXII-1.)

17. The meaning of the negative exponent was known in the beginning, by less than half of the pupils and forgotten by practically the whole group. There was only one question containing a negative exponent, in the whole test and that was 3⁻²; the numerical base, probably, added another difficulty. (See Table VII, p. 31, XXIII.)

18. The number work in radicals was done very well. (See Table VII, pp. 32, 33, XXIX-1; XXX-1, 2; XXXI-1; XXXII-2; XXXIII-2.) As soon, however, as fractional radicands were introduced, the efficiency was decreased about 50%. (See Table VII, pp. 33, XXXIV-1.)

19. In addition and subtraction of surds, the fact that the radicand and the index remained unchanged was known by 71% of the group, and remembered by slightly more than 50% of the whole group. The adding of the coefficients of the radicands was badly done, due to the fact that one coefficient was unity, and hence was absolutely disregarded, by at least 50% of the pupils. (See Table VII, p. 33, XXXV-1, 2.)

20. The elimination of one unknown, by substitution, in a pair of equations, where one equation was linear and the other quadratic, was known by 44% of the pupils on the initial test and remembered by less than 5% of them. (See Table VII, p. 35, XLII-1, 2; XLIII-1, 2.)

TABLE VII

THE ANALYSIS OF TEST II* INTO ONE HUNDRED NINE STEPS, WITH THE NUMBER OF PUPILS GETTING EACH STEP CORRECT, ON THE FOUR TRIALS, FOR GROUP B AND C AND FOR THE TWO TRIALS FOR GROUP D.

Example	Analysis	Steps	No. of Pupils Getting the Indicated Step Correct						
			Group B and C N = 142				Group D N = 45		
			†T ₀	T ₈	T ₁₆	All 4 times	T ₀	T ₉	Both times
I. $\frac{A}{B} = \frac{5}{3}$ $\frac{60}{B} = \frac{5}{3}$ $\frac{5B}{B} = \frac{180}{3}$ B = 36 All 4 Steps Correct	Translation from English into an algebraic equation	1	88	100	105	104	41	20	32
		2	85	100	105	102	41	20	32
	Clearing of fractions Equals ÷ equals	3	85	101	103	103	41	20	32
		4	86	100	101	101	41	20	32
II. $\frac{X}{9} = \frac{5}{3}$ $\frac{5X}{9} = 27$ X = 54 Both Steps Correct	Clearing of fractions	1	123	97	106	106	59	41	33
		2	111	88	106	96	49	23	23
	Equals ÷ equals		110	86	105	94	49	23	23
									14

* See Test II, page 5.

† The subscript gives the number of months between the conclusion of elementary algebra and the giving of the test.

[illegible]

XI.	$\frac{1}{x} + \frac{1}{y} = 7$	1	104	83	81	68	40	38	25	23
	$\frac{1}{x} - \frac{1}{y} = 1$									
	$\frac{2}{x} - \frac{8}{y} = 8$									
XII.	$3x - 2y = 12$	1	92	93	88	71	37	44	34	34
	When $x = 0$, then $y = -6$									
XIII.	$3x - 2y = 12$	1	78	86	76	56	28	38	12	12
	When $y = -3$, then $x = 2$	1	89	75	94	60	34	38	23	22
XIV.	$2x$	2	109	70	98	90	46	42	28	28
	$y + 5$									
	$\frac{2x}{y+5} = \frac{7}{8}$	3	119	86	104	87	59	43	27	27
	$\frac{y+5}{x-1}$	4	90	68	85	67	26	39	25	24
	$x - 1$	5	106	77	96	77	47	42	27	27
	$y + 7$									
	$\frac{x-1}{y+7} = \frac{1}{3}$	6	119	86	101	83	54	43	28	28
	$y + 7$									
	All 6 Steps Correct									
XV.	$2x + 2y = 7$	1	70	50	70	51	15	35	20	1
	$\frac{2x}{3}$	2	40	30	31	21	10	18	3	
	$y = \frac{2x}{3} + 1$	3	43	31	36	18	7	18	6	4
	All 3 Steps Correct									
XVI.	$A^m \cdot A^m = A^{2m}$									
	Multiplication with literal exponents	1	100	77	61	82	41	25	12	12

TABLE VII.—Continued.

The Analysis of Test II into One Hundred Nine Steps, with the Number of Pupils Getting Each Step Correct, on the Four Trials, for Group B and C and for the Two Trials for Group D.—Continued.

Example	Analysis	Steps	No. of Pupils Getting the Indicated Step Correct					
			Group B and C N = 142				Group D N = 45	
			T ₀	T ₈	T ₁₆	All 4 times	T ₀	T ₈
XVII. $\left(-\frac{2}{3}ax^2\right)^3$ 8 27 a ³ x ⁶ All 5 Steps Correct	A negative quantity raised to an odd power	1	101	77	72	78	23	12
		2	96	61	45	48	22	6
		3	94	60	45	46	23	6
		4	104	70	65	78	28	10
		5	104	56	56	60	15	6
XVIII. $\frac{x^{4c}}{x^{2c}} = x^{2c}$ All 5 Steps Correct	Division with literal exponents		70	29	15	18	10	0
		1	77	80	90	87	24	11
XIX. $\sqrt{\frac{25a^2}{36x^4}}$ $\pm$ $\left\{\frac{5}{6}\right\}$ a x ² All 5 Steps Correct	Extracting an even root	1	11	1	0	0	0	0
		2	111	93	94	100	34	21
		3	113	89	91	64	36	22
		4	106	70	85	111	34	22
		5	120	87	91	92	32	21
			11	1	0	0	0	0

XX. $\frac{8\sqrt{15}}{2\sqrt{5}} =$ 4 $\sqrt{3}$ Both Steps Correct	1 2 103 105 101	62 64 62	58 59 57	52 54 52	23 23 22	31 31 31	7 7 7
XXI. $\sqrt{15129}$ 123 I 22 $\frac{51}{44}$ 243 $\frac{729}{729}$ All 3 Steps Correct	1 2 3 32 1	33 22 33 23 133	25 25 23 23 98	28 26 23 23 87	6 6 4 4 59	25 24 23 23 40	2 0 0 0 22
XXII. $x^0 = 1$							
XXIII. 3^{-2} I 3 ² $\frac{1}{9}$ Both Steps Correct	1 2 1 2 1	65 22 22 22 78	7 5 5 4 6	7 4 30 4 8	2 0 0 0 2	34 34 34 34 34	4 4 4 4 3
XXIV. $8^{\frac{1}{3}} = \sqrt[3]{8^2} =$ $(2^3)^{\frac{1}{3}} =$ or $\sqrt[3]{64} =$ 2 ² =	1 2 3	78 38 24	6 2 1	8 6 6	2 0 0	34 33 32	3 1 1
XXV. $x^{\frac{4}{3}} = \sqrt[4]{x^3}$ All 3 Steps Correct		24	1	6	0	32	1
XXVI. $x^{\frac{1}{3}} = \sqrt[3]{x}$ $x^{\frac{1}{3}} = \sqrt[3]{x^5}$	1 1	127 118	95 82	101 77	55 34	43 44	16 9

‡ In this work with exponents, the scores for the fourth trial were better than those of the second or third, for group B and C. This was due to the work of the 66 pupils who were continuing their work in mathematics. See Chapter IV, page 36.

TABLE VII.—Continued.

The Analysis of Test II into One Hundred Nine Steps, with the Number of Pupils Getting Each Step Correct, on the Four Trials, for Group B and C and for the Two Trials for Group D.—Continued.

Example	Analysis	Steps	No. of Pupils Getting the Indicated Step Correct					
			Group B and C N = 142				Group D N = 45	
			T ₀	T ₅	T ₈	T ₁₆	All 4 times	Both times
XXVII. $2x^3 = 2\sqrt{x^2}$	2 is not affected by the exponent Meaning of fractional exponent	1	46	35	24	†51	5	6
XXVIII. $\frac{x^3y^4}{x^2y^3} = \sqrt[3]{xy^2}$	Both Steps Correct	2	120	81	69	94	36	14
			42	35	24	51	5	5
XXIX. $\sqrt{\frac{75}{25x^3}}$	Meaning of fractional exponent Separating radicand into two factors, one of which is a perfect square	1	120	78	78	†81	78	15
		1	132	85	87	93	56	15
XXX. $\sqrt{\frac{18}{49}}$	Perfect square in the numerator and in the denominator	1	94	64	72	75	32	6
$\sqrt{\frac{9}{49} \times 2}$	Both Steps Correct	2	113	46	42	50	14	6
			60	34	31	58	14	4

XXXI. $\sqrt{8a^3b^3} = \sqrt{4a^2b^2} \sqrt{2a}$	1	114	72	72	77	35	33	12	12
XXXII. $\sqrt{32} = \sqrt{16 \times 2} = \pm 4\sqrt{2}$	2	106	50	50	61	22	29	10	8
XXXIII. $\sqrt{\frac{7}{9}} = \sqrt{\frac{1}{9} \times 7} = \pm \frac{1}{3}\sqrt{7}$	3	99	47	47	49	18	30	5	5
XXXIV. $\sqrt{\frac{7}{4}} = 2\sqrt{\frac{1}{4} \times 7} = 2\left(\pm \frac{1}{2}\sqrt{7}\right) = \pm\sqrt{7}$	1	89	40	39	40	15	26	5	0
XXXV. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	1	18	1	1	0	0	0	0	0
XXXVI. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	2	113	67	73	74	67	42	18	18
XXXVII. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	1	18	1	1	0	0	0	0	0
XXXVIII. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	1	21	4	3	3	1	0	0	0
XXXIX. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	2	125	76	65	72	35	40	15	15
XL. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	2	16	4	3	3	1	0	0	0
XLI. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	1	73	34	33	45	13	31	13	13
XLII. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	2	73	36	29	38	13	30	13	13
XLIII. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$		73	34	29	33	13	28	4	4
XLIV. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	1	62	34	27	24	3	18	5	5
XLV. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	2	101	74	73	71	33	42	10	8
XLVI. $\sqrt{2ab} - 3\sqrt{2ab} + \frac{5}{2}\sqrt{2ab} = \frac{1}{2}\sqrt{2ab}$	2	54	34	22	24	3	18	3	3

The factors 4×2 The factors $a^2 \times a$ The factor b^3

The double sign

The square root of 16

The double sign

The square root of 4

The square root of 4

Simplifying

Combining the coefficients

Keeping the same index and radicand

TABLE VII.—Continued.
The Analysis of Test II into One Hundred Nine Steps, with the Number of Pupils Getting Each Step Correct, on the Four Trials, for Group B and C and for the Two Trials for Group D.—Continued.

Example	Analysis	Steps	No. of Pupils Getting the Indicated Step Correct						
			Group B and C N = 142				Group D N = 45		
			T ₀	T ₈	T ₁₆	All 4 times	T ₀	T ₈	Both times
XXXXVI. $(5 - \sqrt{3})(5 + \sqrt{3}) = 25 - \sqrt{9} = 22$ Both Steps Correct	Multiplying Simplifying	1 2	99 47	63 11	80 9	16 3	19 19	8 3	8 3
XXXXVII. $x^2 - 9x = -20$ $(x-5)(x-4) = 0$ $x-5=0$ $x-4=0$ $x=5$ $x=4$ All 6 Steps Correct	Transposing Factoring Equating each factor to zero Transposing	1 2 3 4 5 6	91 91 82 85 81 76	38 36 11 11 11 10	29 27 11 13 14 9	11 9 2 1 1 1	42 40 37 37 37 37	17 16 3 3 3 3	16 16 3 3 3 3
XXXXVIII. $x^2 - 5x - 8 = 8$ $x^2 - 5x = 0$ $x(x-5) = 0$ $x = 0$ $x - 5 = 0$ $x = -5$ All 5 Steps Correct	Transposing Factoring Equating each factor to zero Transposing	1 2 3 4 5	45 28 22 24 21	17 3 3 3 4	16 11 7 6 5	3 3 3 3 2	21 8 5 6 5	7 3 1 1 1	6 1 1 0 0
XXXXIX. $x^2 - 4x - 9 = 0$ $x^2 - 4x = 9$ $x^2 - 4x + 4 = 9 + 4$ Both Steps Correct	Transposing Completing the square	1 2	73 66	6 2	7 0	10 1	43 42	5 2	5 2
			62	2	0	0	42	2	2

XL. $x^2 + 5x - 24 = 0$ $x^2 + 5x = 24$ $x^2 + 5x + \frac{25}{4} = 24 + \frac{25}{4}$ Both Steps Correct	Transposing Completing the square	1	66	4	3	3	0	42	40	40
XLII. $2x^2 + 5x - 3 = 0$ $2x^2 + 5x = 3$ $x^2 + \frac{5}{2}x = \frac{3}{2}$ $x^2 + \frac{5}{2}x + \frac{25}{16} = \frac{3}{2} + \frac{25}{16}$ All 3 Steps Correct	Transposing Dividing by coefficient of x^2 Completing the square	1 2 3	50 32 26	3 0 0	2 0 0	5 0 0	0 0 0	38 31 30	5 2 1	5
XLIII. $x^2 + y^2 = 145$, $x - y = 1$ $x = 1 + y$ $(1 + y)^2 + y^2 = 145$ Both Steps Correct	Transposing Substituting	1 2	63 37	8 3	7 3	6 3	5 1	36 29	1 0	1
XLIV. $xy = -91$, $x + y = -6$ $x = -y - 6$ $(-y - 6)^2 + y^2 = 81$ Both Steps Correct	Transposing Substituting	1 2	55 28	8 2	6 4	5 5	5 2	34 31	0 0	0
XLV. $x^2 + y^2 = 25$ Circle $x, x + 1$ $x(x + 1) = 552$ Both Steps Correct	Knowledge of the graph of this equation Translation	1 1 2	46 66 47	40 31 28	49 33 33	48 40 29	17 12 5	35 35 9	24 6 4	24
XLVI. $8x^2 + 2$ $8x^2 + 2 = 10x$ All 3 Steps Correct	Translating from Eng- lish into an algebraic equation	1 2 3	90 63 92	67 63 57	17 54 23	3 27 52	3 26 22	38 38 35	21 20 15	18
XLVII. $x + \frac{x}{I} = 50$ Ans. No.	Meaning of the word reciprocal	1	84	52	22	50	20	35	15	11
			56	50	47	38	9	35	6	5

CHAPTER IV

THE RELATIONSHIP OF THE AMOUNTS OF ELEMENTARY ALGEBRA REMEMBERED, AFTER VARYING INTERVALS OF TIME, IN TERMS OF GROSS SCORES ON THE FOUR TRIALS OF TEST I AND TEST II.

From a statistical study of the scores of the four trials of Test I, in their relation to each other, the writer found that all the coefficients of correlation were low, ranging from .179 to .549 with an average of .355. The highest one was between the third and fourth trials of the test. $R_{T_9T_{15}} = .549$. This was to have been expected, however, for the knowledge that had been retained through the nine-month interval, seemed to have been well fixed in the pupils' minds. The scores on the first trial of Test I had a lower correlation with those of the second trial than with either those of the third or the fourth trials. This showed that the greatest amount of forgetting took place in the initial three-month interval and that, thereafter, the forgetting was more gradual. Also, after the first forgetting, the pupils tended to approximate their original rank: These findings corresponded with those of Myers¹, that "There is a decided gain in the final recall as the result of intervening recalls."

Test I was to have tested complete mastery of the principles of elementary algebra, but the results showed that the mastery of processes of several steps could not be thus tested. After an interval of three months, some of the very first steps had been forgotten, hence the succeeding steps were not reached. This fact accounted for the low correlations between the gross scores on the first trial with those of the second, third, and fourth trials of Test I.

TABLE VIII

INTERCORRELATIONS OF THE AMOUNTS OF ELEMENTARY ALGEBRA REMEMBERED AFTER INTERVALS OF THREE, NINE, AND FIFTEEN MONTHS

Group A		Test I	
*	T_3	T_9	T_{15}
T_0	.179	.324	.317
T_3		.277	.484
T_9			.549

* This table should be read thus: the correlation of the first trial (t_0) with the trial after 3 mo. (t_3) = .179

¹ Myers, G. C.: Recall in Relation to Retention. *Journal of Education Psychology*, Vol. 5, 1914.

From a statistical study of the scores of the four trials of Test II for group B and C, the writer found some quite significant correlations. (See Table IX.) The correlations of the first trial with those after five, eight, and sixteen months were .694, .639, and .691 respectively. These correlations were very stable; they showed that the pupils held their relative positions very well.

The correlations of the second trial with the third and fourth trials were more significant still; they were .761 and .719 respectively. These showed that those pupils who had remembered more at the end of the five-month interval continued to remember more during the other intervals.

The correlation between the third and fourth trials was not quite as high as the correlations with the second trial; it was .684. This, however, was a significant correlation. The reason it was a little lower was due to the fact that the sixty-six pupils who were continuing their work in mathematics made better scores on the fourth trial than the 76 pupils who had discontinued this work. When these correlations were compared with those of group D it was found that a nine-month interval had caused a much lower correlation. For group D the correlation of the two trials was .470.

TABLE IX
INTERCORRELATIONS OF THE AMOUNTS OF ELEMENTARY ALGEBRA
REMEMBERED AFTER INTERVALS OF FIVE, EIGHT, AND
SIXTEEN MONTHS

Group B and C N=142		(Test II)	
*	T ₅	T ₈	T ₁₆
T ₀	.694	.639	.691
T ₅		.761	.719
T ₈			.684

* This table should be read thus: the correlation of the first trial of the test (t₀) with the trial after five months (t₅) = .694.

CHAPTER V

THE INFLUENCE OF SEX, CHRONOLOGICAL AGE AND INTELLIGENCE UPON THE RETENTION OF ELEMENTARY ALGEBRA.

A. THE INFLUENCE OF SEX UPON THE RETENTION OF ELEMENTARY ALGEBRA

In comparing the boys' scores on Test II with those of the girls', it was found that the mean scores on the first trial were 60.6 ± 2.16 and 58.03 ± 1.59 respectively; on the second 38.7 ± 2.06 and 39.57 ± 1.41 ; on the third 39.2 ± 2.01 and 39.93 ± 1.49 ; and on the fourth 40.44 ± 2.19 and 40.11 ± 1.54 . The sigmas of the differences of the averages for the four trials of the test were 2.68, 2.49, 2.49, and 2.68 respectively. Hence the differences in these mean scores were very insignificant.

When the sigmas of the two groups, on the four trials of the test were compared it was found that the boy's scores showed a little greater variation than those of the girls'. On the first trial, the sigmas for the boys and girls were 15.57 ± 1.53 and 15.07 ± 1.13 respectively; on the second 14.80 ± 1.45 and 13.42 ± 1.00 ; on the third 14.41 ± 1.41 and 14.16 ± 1.57 ; and on the fourth¹ 15.78 ± 1.55 and 14.70 ± 1.10 . Also the coefficients of variation for the two groups were, on the first trial, 25.7 and 25.1 respectively; on the second trial, 38. and 33.9; on the third trial, 36.7 and 35.5; and on the fourth trial, 39.02 and 36.7. Thus differences in these variations were hardly worthy of note.

From a scrutiny of the table of the distributions of scores on the four trials of Test II for the boys and the girls it was concluded that the variations within the sexes was much greater than the variations between the sexes.

B. THE INFLUENCE OF CHRONOLOGICAL AGE UPON RETENTION OF ELEMENTARY ALGEBRA

Chronological age when correlated with the four trials of Test I gave the following coefficients; — .244, +.123, +.175, and +.140, respectively; and with the four trials of Test II, —.401, —.360, —.362, —.334.

¹ Garrett, H. E., Statistics in Psychology and Education. Longmans, Green and Co. 1926. p. 41.

The range of these coefficients was from $-.401$ to $+.175$ with an average of $-.199$.

When the effect of chronological age was held constant the coefficient of correlation between the first and second trials of Test II was lowered about $+.057$. (See Table XVIII, p. 47.)

From these correlations the conclusion was drawn that the younger pupils remembered a little better than the older ones. However, they were the brighter pupils who had reached the grade at an earlier age.

TABLE X

DISTRIBUTION OF SCORES OF THE FOUR TRIALS OF TEST II FOR GROUP B AND C

Boys N = 52 Girls N = 90	T ₀		T ₅		T ₁		T ₁₆	
Scores	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
100-109	0	2						
90-100	5	10			0	1	0	1
80-90	8	11						
70-80	17	17	2	2	2	2	5	0
60-70	16	18	1	6	5	4	10	10
50-60	4	21	8	15	6	13	7	14
40-50	0	10	12	23	11	23	11	11
30-40	2	1	14	24	16	17	5	29
20-30			12	13	9	24	7	15
10-20			2	7	3	6	7	10
0-10			1	0				
Meanscores	60.6	58.03	38.7	39.57	39.2	39.93	40.44	40.11
σ	15.57	15.07	14.8	13.42	14.41	14.16	15.78	14.70

C. THE INFLUENCE OF INTELLIGENCE UPON THE RETENTION OF ELEMENTARY ALGEBRA

The intelligence scores of group A had a fair correlation with those of the first trial of Test I and low positive coefficients with the other three trials. These coefficients ranged from $+.156$ to $+.345$, with an average of $+.204$. The coefficients of the correlation of intelligence with the four trials of Test I were $+.345$, $+.129$, $+.187$, and $+.156$ respectively.

When Test II was used, intelligence correlated positively with all four trials and the coefficients were higher than were

with those with Test I. The range this time was from .395 to .418, with an average of .403. The coefficients of the correlation of intelligence with the four trials of Test II were .399, .401, .395, and .418 respectively.

The reason for these low coefficients was, that, so far, the factors of interest, effort, and purpose had been left out of this discussion. Chapter VI which will deal with these three factors will clearly demonstrate this saying:—What will it profit a pupil, even if he has a good mind, if he is not interested in his work, does not put forth the proper effort, and has no definite purpose in view?

CHAPTER VI

THE EFFECT OF PURPOSE, INTEREST OR SUBJECT PREFERENCE AND EFFORT ON THE RETENTION OF ELEMENTARY ALGEBRA

Let us now consider the effect of purpose, interest or subject preference, and effort on the retention of elementary algebra. A comparison was made between group B and group C with reference to intelligence, interest or subject preference, effort, and memory of elementary algebra. It was found that although the two groups were approximately equivalent from the standpoint of intelligence, that group B excelled group C in its memory of elementary algebra. A detailed analysis of these two groups was next studied, followed by conclusions drawn from the statistical analysis of the scores of group B and group C.

A. COMPARISON OF GROUP B AND C

1. *Intelligence*

The pupils of group B and C divided themselves by their purpose to continue or discontinue the work in mathematics. Group B was composed of sixty-six pupils who continued, and group C had seventy-six pupils who discontinued this work. From the viewpoint of intelligence, group B was approximately equivalent to group C, for although the mean score was 1.88 better, the range was about the same.

TABLE XI

THE PER CENTS OF PUPILS OF GROUP B AND GROUP C WHO HAD THE
INDICATED SCORES ON THE OTIS TESTS.

	Group B N = 66	Group C N = 76
60-70	7.5	1.3
50-60	10.5	24.2
40-50	46.1	44.0
30-40	35.5	21.1
20-30	0.4	9.4
Mean	46.0	44.12
σ	8.67	7.6

2. *Interest*

In their interest, as measured by their subject preference, group B was very superior to group C; 55% of group B

gave mathematics as their first or second choice, as compared with 38.3% of group C.

TABLE XII

THE PER CENTS OF PUPILS OF GROUP B AND GROUP C WHO HAD THE INDICATED SCORES IN INTEREST

Scores	Group B N=66	Group C N=76
90-100	46.9	17.1
80-90	18.1	21.2
70-80	10.7	27.4
60-70	10.7	11.9
50-60	9.1	14.5
40-50	3.0	5.3
30-40	1.5	2.6
Mean	80.83	70.62
σ	18.3	16.8

3. *Effort*

In effort, group B was also superior to group C; 72% of group B had scores above 80 while group C had only 45.8%. Group C had much greater variation in its effort than group B, as shown by the sigmas of the groups.

TABLE XIII

THE PER CENTS OF PUPILS OF GROUP B AND GROUP C WHO HAD THE INDICATED SCORES IN EFFORT

Scores	Group B N=66	Group C N=76
90-100	16.0	17.0
80-90	56.0	28.8
70-80	22.0	19.8
60-70	4.5	18.5
50-60	1.5	10.6
40-50		1.3
30-40		4.0
Mean	83.08	73.22
σ	6.6	15.02

4. *Memory of Elementary Algebra*

When these two groups were compared with reference to the amount of elementary algebra they had remembered, group B was superior to group C. They started out with a mean score that was 7.9 higher than that of group C and they lost 31.7%, 30.7%, and 29.6% as compared with 34.4%, 32.7%, and 31.6% lost by group C.

TABLE XIV

MEAN SCORES ON TEST II AFTER ZERO, FIVE, EIGHT, AND SIXTEEN MONTHS

	T_0	T_5	T_8	T_{16}
Group B	61.8	42.18	42.85	43.47
Group C	53.9	35.37	36.29	36.88
Group B (N=66) σ	14.6	13.0	13.3	13.6
Group C (N=76) σ	15.5	14.26	15.0	9.8

The curves of retention for group B and C, group B, and group C, all three showed that the initial loss was, by far, the greatest. These when plotted gave practically three parallel curves, with that of group B above, and that of group C below the curve of group B and C.



This looked as if these curves might be variations of a general curve, which could be stated by a mathematical formula, as suggested by C. W. Luh,¹ or that they might be variations of some ideal curve as suggested by Strong² and Bean.³

From an examination of the distribution tables of the scores of the two groups, on the four trials of the test, it

¹ Luh, C. W.: The Conditions of Retention. Psychological Monographs, Vol. 31, No. 3, 1922.

² Strong, E. K., Jr.: The Effect of Time-Interval upon Recognition Memory. Psychological Review. Vol. 30, 1913.

³ Beam, C. H.: The Curve of Forgetting. Archives of Psychology, No. 21, 1912.

was found that group B has 33.3% above 80 and 3.3% below 40, on the first trial; while group C had 18.4% above 80 and 14.5% below 40. On the second trial, group B had 30.3% above 60 and 18.3% below 40; while group C had 15.8% above 60 and 32.9% below 40. On the third trial group B had 30.3% above 60 and 13.6% below 40; while group C had 14.5% above 60 and 36.9% below 40. On the fourth trial, group B had 57.6% above 60 and 3.3% below 40; while group C had 13.1% above 60 and 47.3% below 40.

TABLE XV

DISTRIBUTION TABLES OF THE SCORES OF THE FOUR TRIALS OF TEST II,
FOR GROUP B AND GROUP C

Group B, N=66

Group C, N=76..

Scores	T ₀		T ₅		T ₈		T ₁₈	
	B	C	B	C	B	C	B	C
100-109		2				1		
90-100	9	6						
80-90	13	6	1	2		3	4	1
70-80	15	18	3	4	6	2	17	2
60-70	13	22	16	6	14	5	17	7
50-60	14	11	17	17	18	17	13	9
40-50	1	9	18	22	19	20	13	21
30-40	1	2	9	17	6	21	1	11
20-30			1	8		7	1	15
10-20			1		3			10
0-10								
Mean	62.1	56.3	42.17	36.69	42.73	37.57	52.46	34.9
σ	14.28	15.86	12.99	14.25	13.07	10.86	13.58	9.87

B. CONCLUSIONS FROM THE ANALYSIS OF THE SCORES OF GROUP B AND GROUP C

1. Simple Correlations

When the correlations of the first with the other three trials of Test II, for group B and group C were compared, it was found that the correlations for the first group increased while those for the second group decreased. The coefficients for group B were .577, and .591, and .776 respectively, while those of group C were .765, .661, and .540 respectively. (See Tables XVI and XVII, pp. 45, 46.)

When the correlations of the second trial with the third and fourth trials were compared, the difference was still greater, for group B, .838 and .752 respectively, and for group C, .691 and .648 respectively; and for the correlations of the third and fourth trials the difference was very great indeed, .776 for group B and .564 for group C. The fact that group B intended to continue the work in mathematics seemed to steady their retention. These findings confirm Meumann's¹ statement that "we remember only that which we have apprehended attentively, with the aim of remembering it."

TABLE XVI

SIMPLE CORRELATIONS BETWEEN THE FOUR TRIALS OF TEST II, CHRONOLOGICAL AGE, INTELLIGENCE, SUBJECT PREFERENCE OR INTEREST, AND EFFORT

Group B N=66	T ₅ *	T ₈	T ₁₆	CA	I	SP	E
T ₀	.577	.591	.776	-.442	.613	.416	.644
T ₅		.838	.752	-.356	.319	.375	.487
T ₈			.776	-.446	.517	.493	.726
T ₁₆				-.564	.838	.495	.948
CA					-.712	-.148	-.647
†I						.157	.519
SP							.431

* The subscript=number of months from the completion of elementary algebra until the time the test was given.

†Intelligence was determined by the Otia Self-Administering Tests of Mental Ability. Higher Examination—Form A.

The intelligence scores gave higher correlations for group B than for group C. For group B, the range was from .319 to .838 with an average of .572, and for group C .346 to .522 with an average of .433. (See Table XVI above and Table XVII, p. 46.)

The interest or subject preference scores also gave higher correlations for group B than for group C. The coefficients of correlations for group B ranged from .375 to .495 with an average of .445 while those of group C ranged from .174 to .334 with an average of .215. This

¹ Meumann, E.: *The Psychology of Learning*, translated by J. W. Baird, D. Appleton and Company, 1913.

fact showed clearly that as the interest waned the relationship weakened. (See Table XVI, p. 45, and Table XVII, below.)

The correlations with the effort scores emphasized still further the growing difference between the retention of the two groups. Probably the most striking difference was found in the correlation between the fourth trial of the test and the effort scores. For group B, this was .948 and for group C .290. Again these correlations showed that the mind set of the pupils played a great part in their retention: group B had apprehended attentively with the aim of remembering what they had studied. (See Tables XVI, XVII.)

TABLE XVII

SIMPLE CORRELATIONS BETWEEN THE FOUR TRIALS OF TEST II, CHRONOLOGICAL AGE, INTELLIGENCE, SUBJECT PREFERENCE OR INTEREST AND EFFORT.

Group C N=76	T ₅	T ₈	T ₁₆	CA	I	SP	E
T ₀	.765	.661	.540	-.433	.401	.334	.536
T ₅		.691	.648	-.405	.522	.162	.565
T ₈			.564	-.359	.464	.174	.406
T ₁₆				-.268	.346	.191	.290
CA					-.309	-.117	-.509
I						.117	.289
SP							.345

2. Partial Correlations

The simple correlations between the first and second trials of Test II were .577, and .765 for group B and C, respectively. When the effect of chronological age was partialled out, the correlations from groups B and C were lowered .076 and .049 respectively. When the effect of intelligence was partialled out, the correlations were lowered .081 and .119 respectively. When the effect of subject preference or interest was partialled out, the correlations were lowered .077 and .000 respectively. When the effect of effort was partialled out, the correlations were lowered .183 and .101 respectively. (See Table XVIII, p. 47.)

When the effect of subject preference or interest was partialled out (see Table XVIII), group B's coefficient was lowered .077, while that of group C was not changed. When the effect of effort was held constant, group B's

coefficient was lowered .082 more than that of group C. These two results showed clearly that subject preference or interest, and effort were two potent factors in the retention of these principles of elementary algebra.

TABLE XVIII

PARTIAL CORRELATIONS OF SCORES OF THE FIRST TWO TRIALS OF TEST II, WITH THE EFFECT OF (A) CHRONOLOGICAL AGE, (B) INTELLIGENCE, (C) SUBJECT PREFERENCE OR INTEREST, (D) EFFORT HELD CONSTANT

	Group B N=66	Group C N=76
A.....	.501	.716
B.....	.496	.646
C.....	.500	.765
D.....	.394	.664
Simple Correlation	.577	.765

3. Multiple Correlations

When the influence of subject preference or interest, and effort was added by means of the statistical device of multiple correlation, group B gained much more than group C. This was shown particularly in the multiple correlations of the scores after a sixteen month interval. In this instance, group B's coefficient was .953, as compared with .364 for group C.

TABLE XIX

MULTIPLE CORRELATIONS OF THE SCORES OF THE FOUR TRIALS OF TEST II WITH THE INFLUENCE OF SUBJECT PREFERENCE, OR INTEREST, AND EFFORT ADDED

	Group B N=66	Group C N=76
*R T_0 SP and E	.662	.559
R T_5 SP and E	.519	.569
R T_8 SP and E	.753	.407
R T_{16} SP and E	.953	.364

* This table should be read: The multiple correlation of the scores on Test II, given after an interval of zero months, from the completion of elementary algebra, with the influence of subject preference and effort added.

Table XIX showed clearly that, as the time interval increased, the effect of interest and effort became very marked; group B's correlations increased, while group C's decreased. These results demonstrated emphatically that interest and effort were two important factors in retention.

The fact that group B forged ahead of group C throughout the whole discussion, in this chapter, showed the dynamic influence of the purpose of the first group. The three factors, interest, effort, and purpose, helped the retention of group B and their absence hindered the retention of group C.

CHAPTER VII

SUMMARY AND CONCLUSIONS

A. COMPARISON OF TEST I AND TEST II AS MEASURING SCALES FOR RETENTION

Test I was not a good measuring scale for retention, for several reasons. One was, that it did not completely measure the group, for some perfect scores were made and also some zero scores. A second reason was, that Test I required more recall than recognition and hence, was very difficult to solve, after a lapse of time. A third reason was, that the pupils were expected to solve a problem from start to finish, and some times, many consecutive steps were involved, each one of which was dependent upon the preceding step. If the pupil could not do one of these steps, he failed to get the succeeding ones.

However, Test I was more typical of the kind of tests in mathematics, that the pupils would be asked to pass, later on, in high school and college. The writer thought and still thinks, that tests similar to Test I were and are needed to test complete mastery of the various topics of the subject being studied.

Test II was a better measuring scale for retention than Test I, for several reasons. One was, that it did measure the group, no one ever made a perfect score, and only once, was a zero score made. A second reason was, that it was composed of many examples of few steps each, so that recall was easier than in Test I. A third reason was, that it diagnosed the pupils' difficulties, more readily, than did Test I.

The writer felt that both tests were needed, Test II to diagnose individual difficulties and Test I to test complete mastery of the principles of elementary algebra.

B. WHICH TOPICS WERE FORGOTTEN

Mechanical processes of several steps, such as the solution of a quadratic equation, in one unknown by the method of completing the square, were quickly forgotten. Such other processes as extracting the square root of an integer of many digits; the solution of simultaneous equations, where one was linear and the other quadratic; the work with irrational equations; and the meaning of negative and fractional exponents, when used with numbers for bases;

were forgotten almost completely during a five month interval. (See Tables VI and VII, pp. 16, 26.)

C. WHICH TOPICS WERE REMEMBERED?

The topics which were remembered were the following: Translation from an English sentence into a mathematical statement; multiplication and division of monomials having literal bases, this was true even when the exponents were literal; the extracting of the root of a monomial; the simplification of the square root of an integer that contained a factor that was a perfect square; the meaning of the zeroth exponent; the meaning of the fractional exponent, especially when the base was a letter; and the handling of simultaneous linear equations. (See Tables VI and VII, pp. 16, 26.)

D. DID THE FOLLOWING FACTORS PLAY ANY PART IN THE REMEMBERING?

1. *Chronological Age*

The younger pupils remembered better than the older pupils, as shown by the negative correlations found. These ranged from $-.122$ to $-.564$ with a mean of $-.370$. This was probably due to the fact that they were brighter pupils as they had reached the grade at an earlier age. (See Tables XVI, XVII, pp. 45, 46.)

2. *Intelligence*

Intelligence had a positive correlation with all factors involved in this study except chronological age. With the four trials of Test II the range was from $.346$ to $.838$ with a mean of $.470$. The correlations of intelligence, with the scores on the four trials of Test II made by group B, were by far the most significant. (See Tables XVI, XVII, pp. 45, 46.) As W. D. Tait¹ said, "Impressions that harmonize with the existing attitudes stand a good chance of being remembered," so these sixty-six pupils whose purpose it was to continue with the work in mathematics, used their intelligence when studying and remembered a larger per cent of what they had studied.

3. *Subject Preference or Interest*

This study showed most conclusively the great effect of subject preference or interest on retention. The correlation of subject preference with the scores on the four trials of

¹ Tait, W. D.: The Effect of Psycho-Physical Attitude on Memory. *Journal of Abnormal Psychology*, Vol. 8, pp. 10-37, 1913.

Test II, for group B, ranged from .375 to .495 with a mean of .445; for group C, from .162 to .334 with a mean of .215. Group C did not intend to continue with mathematics and hence had lost interest. (See Tables XVI and XVII, pp. 45, 46.) The partial and multiple correlations emphasized greatly the dynamic effect of interest on retention. (See Tables XVIII, XIX, p. 47.)

4. *Effort*

The correlations of the effort scores with those of Test II for group B ranged from .487 to .948 with a mean of .701 while those for group C ranged from .536 to .290 in a decreasing relationship, with a mean of .449. (See Tables XVI, XVII, pp. 45, 46.) As H. B. Reed¹ said, "High retention may be due to strong imprinting as well as to the presence of an association" so the effort of group B helped their memory.

5. *Sex*

Sex had very little weight upon retention. The girls remembered a little better than the boys, but the girls had more freedom in their choice as to whether to continue or discontinue their work in mathematics. The boys taking the technical course had no choice but to continue. Hence the interest factor for the girls was a little stronger than for the boys. The variations within the sexes was much greater than between the sexes. (See Chapter V, p. 38.)

E. SUMMARY OF CONCLUSIONS

1. Test I, designed to test mastery of algebraic problems of several steps, was not a good instrument for measuring retention.
2. Test II, designed to test individual difficulties, was a good instrument for measuring retention.
3. The translation of English statements into algebraic equations was well remembered.
4. Fundamental principles and processes that had been overlearned by constant practice and application, were well remembered.
5. The work in radicals and surds was quickly forgotten. The writer thinks that this work should have been introduced earlier in the course, so that more overlearning could have taken place.

¹ Reed, H. B.: Associative Aids. *Psychological Review*, pp. 128-155; 257-285, 1908.

6. The solution of quadratic equation in one unknown, by the completing the square method, was also quickly forgotten. This work should have been stressed more.

7. Irrational equations and simultaneous equations where one is linear and the other quadratic were remembered by very few pupils. More drill was needed here.

8. The younger pupils remembered a little better than the older ones.

9. Sex had no influence on retention.

10. The pupils with higher intelligence scores remembered better than those with low scores.

11. Those pupils whose purpose it was to continue studying algebra remembered better than those who knew they intended to discontinue algebra.

12. The pupils who continued the work in algebra, were more interested in it and put forth greater effort than the pupils who discontinued algebra.

F.—FINAL COMMENTS

As G. C. Myers¹ has so well said that "Wrong answers deserve more careful study; in many cases they give more information than the study of correct answers," so the writer would urge teachers of mathematics to study the mistakes of their individual pupils.

From the papers used in this investigation, the conclusion was drawn that carelessness was responsible for a great many of the errors. Pupils who got a step correct three out of four times would make some error to invalidate their work, the other time. Errors like the following $2^{\frac{2}{3}} = \frac{8}{3}$, $2 \times 2^{\frac{2}{3}} = \sqrt[3]{2x}$ might have been carelessness, but then again they might have been a lack of knowledge. The only safe thing to have done, was to have seen the pupils individually, at some conference hour, and to have straightened out, either their bad habits or their lack of knowledge.

Downey² said "the rest period with its break up of certain mechanisms gives freedom for new adaptations" and

¹ Myers, G. C.: Delayed Recall in History. *Journal of Educational Psychology*, Vol. 8, pp. 275-288, 1917.

² Downey, J. E., and Anderson, J. E.: Retention of Skill after Lapses of Practice. *American Journal of Psychology*, Vol. 28, pp. 396-408, 1917.

Book¹ said that "the rest period causes the dropping out of bad habits," so after a reasonable interval test again on the difficulties found in the preceding examination.

C. Guillet² said that "attention, mental imagery, and understanding were the powers that aid us to remember and the ability to grasp logical relations and to see meaning were also considerable factors therein." And W. H. Burnham³ stated that "It takes time for an impression to become so fixed that it can be reproduced after a long interval, for it to become part of a permanent store of memory."

This study has well verified the above statements, associative bond must be formed if the principles of elementary algebra are to be remembered and time must be taken to form these bonds.

The shorter the interval between the conclusion of elementary algebra and the first recall, the greater was the retention. After the first recall, there was very little loss, in some instances there was a slight gain. The general findings of this investigation agreed with those of Ebbinghaus, Radossawijewitsch, Book, Bean, and others. The initial loss was the great loss and then the curve became almost parallel to its axis.

THE FACTORS, PURPOSE, INTEREST, AND EFFORT, were the most potent ones found in this study on the Retention of Elementary Algebra Through Quadratics, after Varying Intervals of Time.

¹ Book, W. F.: *The Psychology of Skill with Special Reference to Its Acquisition in Typewriting*. University of Montana Publications of Psychology, Bulletin No. 53, pp. 1818, 1908.

² Guillet, C.: *A Study of the Memory of Young Women*. *Journal of Educational Psychology*, Vol. 8, pp. 65-84, 1917.

³ Burnham, W. H.: *Retroactive Amnesia*. *American Journal of Psychology*, Vol. 14, pp. 382-386, 1903.

APPENDIX

A. BIBLIOGRAPHY

I. GENERAL REFERENCES

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- GATES, A. J.: Psychology for Students of Education. New York, 1927.
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- WATSON, J. B.: Psychology from the Standpoint of a Behaviorist. Lippincott.

II. REFERENCES ON RETENTION

- ABELSON, A. R.: The measurement of Mental Ability of Backward Children. British Journal of Psychology, Vol. 4, pp. 268-314, 1911.
- In 1911, in eight of the London county schools for defectives, eighty-eight girls and forty-three boys were tested. Correlations between memory for names and intelligence, as measured by ability to perform tasks, were found to be .18 for girls and .19 for boys. Correlation between intelligence and estimated ability in arithmetic was .30 for girls and .32 for boys.
- ACHILLES, E. M.: Experimental Studies in Recall and Recognition. Archives of Psychology, No. 44, 1920.
- A good review of the work of Hollingworth, Bean, McDougall, Myers, and Strong. Achilles states that there is quite a difference between recognition and recall.
- BALLARD, PHILIP BOSWOOD: Obliviscence and Reminiscence. British Journal of Psychology, Monograph Supplements, Vol. 1, pp. 82, 1913.
- Experimental materials used were lines from "Ancient Mariner" and "The Wreck of the Hesperus," with 5,192 elementary school children; "Wynken, Blynken, and Nod," "Meddlesome Matty" and others with children from 5 to 7 years of age.
- BASSETT, SARAH JANET: Retention of History in the Sixth, Seventh, and Eighth Grades with Special Reference to the Factors that Influence Retention. The Johns Hopkins University Press, Baltimore, 1928.

Contains an excellent bibliography. 1229 pupils in sixth, seventh, and eighth grades were tested after intervals of four, eight, twelve, and sixteen months. "The practical suggestion for educational procedure is that history should be presented in a manner to appeal to the various interests of the child and should be rich in associations."

BEAN, C. H.: The Curve of Forgetting. *Archives of Psychology*, No. 21, 1912.

A splendid summary of the work of Ebbinghaus, Radossawljewitch, E. N. Henderson, E. L. Thorndike, and then his own experiment in the Indiana State Normal School to find out: 1. The manner in which students often forget what they learn in class. 2. How students recall under class room conditions.

BELL, J. CARLETON, and MCCOLLUM, D. F.: A Study of the Attainments of Pupils in United States History. *Journal of Educational Psychology*, Vol. 8, 1917.

It is justifiable to consider that ability in responding to tests of historical facts is a fair index of historical ability.

BINET, A., and HENRI, V.: La Mémoire des mots et des phrases. *L'Année psychologique*, Vol. 1, pp. 1-59, 1894.

Forgetting increases rapidly with the length of the phrases and the series of the phrases; for one phrase of twenty words (eight groups of words) about one-eighth, for one series of phrases of eighty words (twenty-four groups of words) about one-third.

BOOK, W. F.: The Psychology of Skill; with Special Reference to its Acquisition in Typewriting. University of Montana Publications of Psychology, Bulletin No. 53, Psychological Series No. I, 1908.

Ten ten-minute practice periods, after a lapse of one and a half years gave an actual gain of one hundred three strokes per day over the record made during the last ten regular tests.

BOSWELL, F. P., and FOSTER, W. S.: On Memorizing with the Intention Permanently to Retain. *American Journal of Psychology*, Vol. 27, pp. 420-429, 1916.

Material used—one hundred twenty-eight pairs of Chinese-English vocabulary arranged in eight series of sixteen pairs each. Subjects—four graduate students. Read aloud for sixteen continuous repetitions. After a five-minute interval the Chinese words were shown in haphazard order and the observers were asked to give the English equivalent. Conclusion: The intent to learn for permanent retention does bring about the desired end, in the case of learning a vocabulary.

BROOKS, F. D.: Changes in Mental Traits with Age, Determined by Annual Re-tests. *Teachers College, Columbia University Contributions to Education*, No. 161, 1921.

Between I. Q. and improvement there is a low, though probably significant positive correlation and this correlation is highest in those tests which measure the higher intellectual capacities.

BROWN, WARNER: To What Extent is Memory Measured by a Single Recall. *Journal of Experimental Psychology*, Vol. 6, pp. 337-382, 1923.

Students in an elementary course in psychology were asked to write in five minutes the names of as many states in the United States as they could remember. One half hour later, without any warning, they repeated the performance. There was a net gain per list of more than three states.

BROWN, W.: Some Experimental Results in the Correlation of Mental Abilities. *British Journal of Psychology*, Vol. 3, pp. 296-322, 1919.

One hundred fourteen adults and one hundred forty-five children between the ages of eleven and twelve years were used. The test

material was syllables and poetry. Retention was tested after an interval of twenty-four hours. Several teachers' gradings were used as the measure of intelligence.

BURNHAM, WILLIAM H.: Retroactive Amnesia: Illustrative Cases and a Tentative Explanation. *American Journal of Psychology*, Vol. 14, pp. 382-396, 1903.

"The fixing of an impression depends upon a physiological process. This is not only making a permanent impression upon the nerve cells but also a process of association, of the organization of the new impression with the old ones."

BOURDON, B.: Influence de l'âge sur la mémoire immédiate. *Review Philosophique*, Vol. 38, pp. 148-167, 1894.

Immediate memory increased a little from eight to twenty years of age, more from eight to fourteen than from fourteen to twenty.

BOURDON, B.: Recherches sur l'habitude. *L'Année psychologique*, Vol. 8, 1901.

Nine different tests were given on marking out letters, pairs of English and French equivalents, English and German equivalents, etc. Most of these experiences show some influence of habit.

COBER, EMANUEL WILSON: A Study of High School Pupils with a View to Determining the Extent of Recollection of Once Familiar Facts. A Thesis, University of Pennsylvania, pp. 46, 1912.

Reviews of once familiar facts seem justified, if such facts are to be of any practical value after learning. One or two reviews at the beginning of the school year might be profitable, followed by occasional reviews at continually increasing intervals. Boys gave better results than girls.

COLVIN, STEPHEN S., and MYERS, E. J.: The Development of Imagination in School Children and the Relation between Ideational Types and Retentivity of Material Appealing to Various Sense Departments. *Psychological Review Monograph Supplements*, Vol. 9, pp. 85-124, 1909.

CROSLAND, HAROLD R.: A Qualitative Analysis of the Process of Forgetting. *The Psychological Monograph*, Vol. 29, No. 1, pp. 159, 1921.

Forgetting is the essential factor of mental evolution, it is the means by which consciousness rids itself of useless and excess lumber, and this active principle of dissociation bears a close similitude to the principle of natural selection in biology.

DALLENBACH, K. M.: The Relation of Memory Error to Time Interval. *Psychological Review*, Vol. 20, pp. 323-337, 1913.

Pictures were used for one experiment and geometrical figures for another. Conclusion: (1) Per cent of error increases with the time interval; (2) Forgetting is very great at the second recall; (3) at the third and last recall the increase is more gradual.

DALLENBACH, K. M.: The Effect of Practice upon Visual Apprehension in School Children. *Journal of Educational Psychology*, Vol. 5, pp. 321-334, 387-404, 1914.

Ten-minute drills, for seventeen weeks, were given to second grade children in rapid observation and reproduction of various visual materials. The nineteen practiced children were superior in recall and description, to their unpracticed school mates.

DE WEERDT, ESTHER H.: The Permanence of Improvement of Fifth Grade Children in Certain Mental Functions. *Journal of Educational Research*, Vol. 16, pp. 127-131, 1927.

Forty-five complete practice records in seven tests were obtained over a period of eleven consecutive school days. Four months after the practice series closed, the seven tests were again given on two

consecutive days to determine the amount of improvement retained over the time of non-specific practice. The two days of re-testing brought the group to an efficiency level, about equal to that, at which they left off.

DOWNEY, J. E., and ANDERSON, J. E.: Retention of Skill after Lapses of Practice. *American Journal of Psychology*, Vol. 28, pp. 396-408, 1917.

The rest interval with its break up of certain mechanisms, gives freedom for new adaptations.

DUGAS, L.: *La Mémoire et l'oubli*. Ernest Fammarrion, Editeur, 26 Rue Racine, Paris, 1917, pp. 356.

Laws of retaining and forgetting are discussed.

EBBINGHAUS, H.: *Memory* (1883), Translated by H. A. Ruger and Clara E. Bussenius. T. C. Bureau of Publication, New York, 1913.

Very great is the dependence of retention and reproduction upon the intensity of the attention and interest. The methods of Natural Science should be used to enlarge our knowledge of memory. With repetition of the syllable series, not only are individual terms associated with their immediate sequents but connections are also made between each term and several of those which follow it, beyond intervening members.

EBBINGHAUS, H.: "Über das Gedächtnis." *Untersuchungen zur experimentellen Psychologie*. Leipzig, Duncker und Humboldt, 1885.

First one to study forgetting experimentally. He was both the observer and experimenter. He first used nonsense syllables and later a stanza from Byron's "Don Juan." His curve of forgetting showed a pronounced drop which was followed by a lessening of the slope until, after a day, its course became almost parallel to its axis.

EIKENBERRY, D. H.: Permanence of High School Learning. *Journal of Educational Psychology*, Vol. 14, pp. 463-81, November, 1926.

There is a positive correlation between test scores and interest in school subjects. There is a positive correlation of about .5 between intelligence as measured by our tests and test scores. There is no correlation between intelligence and school marks. There is a rough correspondence between intelligence and interest in school subjects. Retention was in the following order (1) U. S. History; (2) Ancient History; (3) Geometry; (4) Latin; (5) Chemistry; (6) Physics.

EISENBERG, J. L.: Experimental Studies in Spelling, to Determine whether Continuous Repetition or Repetition after an Interval is more Effective in Memorizing. A Thesis, University of Pennsylvania, pp. 80, 1913.

Conclusion, from investigations lasting from 1907 to 1912, is that both in immediate results and in results at the end of the week, repetition after an interval is more effective in memorizing in spelling than continuous repetition. .93 in favor of repetition after an interval.

FEINGOLD, G. A.: Recognition and Discrimination. *Psychological Review Monograph*, No. 78, 1915.

Recognition ability varies inversely as the number of objects perceived, inversely as the number of objects exposed and directly as the temporal length of the perception. In his experiment only eight words were used each time and the exposure time was short.

FINKENBINDER, N. O.: The Curve of Forgetting. *American Journal of Psychology*, Vol. 24, pp. 8-32, 1913.

One thousand nonsense syllables were learned until the re-agent could anticipate the next syllable. Forgetting was measured by the re-learning method. The process of forgetting was found to be some-

what slower than Ebbinghaus found it, but somewhat faster than Radossawljewitsch found.

FINKENBINDER, N. O.: *The Remembrance of Problems and their Solutions. A Study in Logical Memory. A Dissertation*, Clark University, Worcester, Mass., 1914. Reprinted from the *American Journal of Psychology*, Vol. 25, 1914, pp. 32-81.

"The visual image is the strikingly dominant feature of recall." This experiment embraces, approximately, 200,000 images. Of all his introspective data about 90% was visual imagery.

GARFINKLE, M. A.: *The Effect of the Summer Vacation on Ability in the Fundamentals of Arithmetic. Journal of Educational Psychology*, Vol. 10, pp. 44-47, 1919.

There was a decrease in both speed and accuracy, with the exception of an increase in speed for addition.

GATES, ARTHUR I.: *Correlations and Sex Differences in Memory and Substituting. California University Publications in Psychology*, Vol. I, No. 6, pp. 345-350, 1916.

The correlation for the sexes separately are very similar. The women excel slightly in the memory tests, while the men are somewhat superior in the substitution.

GATES, ARTHUR I.: *Correlations of Immediate and Delayed Recall. Journal of Educational Psychology*, pp. 489-496, 1918.

Nonsense syllables and also connected sense material, in the form of biographies were used with elementary school children, from grades three to eight. Some of the conclusions were that (1) a positive and high correlation exists between immediate and delayed recall, when both functions are measured by absolute amounts; (2) all tests of immediate and delayed recall show a correlation with teachers' estimate of general intelligence with a central tendency of about 0.50; (3) correlation rather than compensation is the rule; (4) the rapid learner is not the rapid forgetter.

GUILLET, C.: *A Study of the Memory of Young Women. Journal of Educational Psychology*, Vol. 8, pp. 65-84, 1917.

"It is evidently not our retentive power that conditions our general ability, nor is it our power of sense imagery, neither is it primarily the effort of attention that makes us efficient; but the ability to grasp logical relations, and to see meanings is a considerable factor therein."

HENDERSON, E. N.: *The Study of Memory for Connected Trains of Thought. Psychological Review Monograph Supplement*, Vol. 5, No. 6, p. 94, 1903.

The Material was not completely learned, as it was read only once, nevertheless, the fast learner who got the most from this one reading, retained the most. Older students do not remember a greater percentage of what they have learned, training would, therefore, seem to affect more the power to learn than the power to retain.

HENMON, V. A. C.: *The Relation between Learning and Retention and the Amount to be Learned. Journal of Experimental Psychology*, Vol. 2, pp. 476-484, 1917.

Materials used were (1) Nonsense syllables, (2) poetry, (3) prose. From his experiments Ebbinghaus' law does not well represent a general tendency.

HENMON, V. A. C.: *The Relation between Mode of Presentation and Retention. Psychological Review*, Vol. 19, pp. 76-96, 1912.

When a comparison is made of the visual and auditory, the superiority of the auditory holds for all materials, nonsense syllables, numbers, or nouns. The visual-auditory method was superior to the visual in 87% of the fifty-four cases but superior to only 46% of the auditory

presentation. The visual-auditory-motor (articulatory) is inferior to visual-auditory in 59%, to the auditory in 55% and to the visual in 15%. All methods of presentation are inferior to the visual alone.

HILL, D. S.: Minor Studies in Learning and Relearning. *Journal of Educational Psychology*, Vol. 5, pp. 377-381, 1914.

First Study was mirror drawing; the second one was a class experiment on periods of learning by use of substitution tests; the third study was concerned with cancellation tests.

HÖFFDING, H.: Zur Theorie der Widererkennen. *Philosophische Studien*, VIII, pp. 86-96, 1893.

The perception of an old object is qualitatively different from the perception of a new object, due to a fusion of the perception with an immediately aroused image of itself. This qualitative difference (Bekanntheitsqualität) is aroused because of an increased ease of nerve functioning due to this fusion.

HOLLINGSWORTH, H. L.: Characteristic Differences between Recall and Recognition. *American Journal of Psychology*, pp. 532-544, 1913.

Nonsense syllables show a considerable loss; words show a smaller loss; and form shows no loss at all.

HUNTER, W. S.: The Delayed Reaction in a Child. *Psychological Review*, Vol. 24, pp. 74-87, 1917.

A child thirteen to sixteen months old, was used. The child was asked to find a top, which he had seen placed, some time previous.

JONES, H. E.: Experimental Studies of College Teaching. The Effect of Examination on Permanence of Learning. *Archives of Psychology*, No. 68, pp. 1-70, 1923.

Lectures on neural functions, reasoning, feeling, etc., were given to college students. Tests were given immediately at the close of the lecture, after intervals of one, three, seven days. The mean coefficient between immediate and delayed recall for four hundred fifty-three cases was .576.

KENNEDY, F.: On the Experimental Investigation of Memory. *Psychological Review*, Vol. 5, pp. 477-499, 1898.

Summarizes some of the worthwhile studies in retention. Concluded that school children's memories become more accurate with age.

KIRBY, T. J.: Practice in the Case of School Children. *Teachers College, Columbia University Contributions to Education*, No. 58, pp. 96, 1913.

In 1911 and 1912—1350 children in the third and fourth grades in New York City were drilled in addition and division. Retention tests were given in June and September 1912 without the previous knowledge of either the children or teachers. In September, in order to regain their standing in June, more than one half of the time was needed for addition and three-fifths for division.

KIRKPATRICK, E. A.: An Experimental Study of Memory. *Psychological Review*, Vol. 1, pp. 602-609, 1894.

The power of recognition seems to be nearly double that of recall. The act of writing has a marked effect, especially upon recognition.

KJERSTAD, C. L.: The Form of the Learning Curves of Memory. *Psychological Review Monograph Supplements*, Vol. 26, No. 116, pp. 89, 1919.

What changes take place in the learning curve when certain factors are varied such as: (1) practice, (2) individual differences, (3) changes in kind of material, (4) changes in length of material, (5) changes in time of presentation, (6) changes in factors like warming up and ennui, which may involve interest, attention, effort, and other factors.

KRAMER, GRACE: Do Children Forget during the Vacation? *Baltimore Bulletin of Education*, Vol. 6, No. 3, December, 1927, pp. 56-60.

"The children as a group proved fairly stable in their retention of arithmetic. In reading ability, the trend is towards appreciable improvement with over 40% of the children."

Lee, A. L.: An Experimental Study of Retention and its Relation to Intelligence. *Psychological Monographs*, Vol. 34, No. 4, pp. 45, 1925.

Pictures of familiar objects, simple nouns, geometrical forms, and nonsense syllables were the materials used with three hundred ten children in grades three to eight, in two New York City public schools. Retention was measured after thirty minutes and twenty-four hours. She concluded that intelligence correlated higher with immediate than delayed recall. The correlation between recognition and recall are most insignificant (.05).

LEHMAN, A.: Über Wiedererkennen. *Philosophische Studien*, V, pp. 96-156, 1889.

Recognition has two meanings in daily life: 1. To recognize, knowing you have seen previously, but not knowing where; 2. To recognize, knowing you have seen previously and knowing where, when, etc.

LUH, C. W.: The Conditions of Retention. *Psychological Monographs*, Vol. 31, No. 3, pp. 87, 1922.

An excellent summary of the works of the prominent investigators of retention. His own experiment had two parts. First, Twenty adults learned series of syllables until they could successfully anticipate each succeeding syllable. His conclusion about retention agreed with earlier investigators, with the exception of Ballard. Second, The degree of learning was varied, this had no considerable effect upon the amount of retention, except that after a lapse of four hours the gross amount of error increases with the degree of learning.

LYON, D. O.: The Relation of Length of Material to Time taken for Learning and the Optimum Distribution of Time. *Journal of Educational Psychology*, Vol. 5, pp. 1-9, 85-91, 155-163, 1913.

Materials used—nonsense syllables, digits, prose and poetry. Methods used: (1) continuous, (2) once and twice per day, once a week. Methods of presentation used—(1) Reading the material to the subject, (2) letting him read it. Conclusions—The total time taken by the once a day method is much less than the continuous method and the material is remembered longer.

LYON, D. O.: Memory and the Learning Process. *Warwick and York, Baltimore, Maryland*, pp. 76-154, 1917.

The relation of quickness of learning to retentiveness depends on many factors, the most important of which are method, material, and interval. Children may have as good or better retentive capacity as adults, but their attentive capacity is inferior.

LYON, D. O.: The Relation of Quickness of Learning to Retentiveness. *Archives of Psychology*, No. 34, pp. 60, 1916.

The reagent's attitude affects his retentiveness. High I. Q.'s learn more quickly and retain longer. High school pupils averaging from sixteen to seventeen years retain more of the poetry than do the younger grammar school pupils and older college students.

MABAI, S.: The Effects of Repetition upon Retention. *Journal of Experimental Psychology*, Vol. 5, pp. 147-151, 1922.

Mabai corroborated the findings of Ebbinghaus, for he found that each three repetitions saved approximately one repetition after twenty-four hours.

MCGEOCH, J. A., and WHITLEY, P. L.: The Recall of Observed Material. *Journal of Educational Psychology*, Vol. 17, pp. 419-425, 1926.

The curve of retention varies with the method of measurement, the forgetting being much greater when measured by narrative reproduction, than it is, when measured by answers to an interrogatory. The curves for both the narrative and interrogatory do not resemble Ebbinghaus's curve of forgetting. It is probable that the difference is due to differences in material.

MCGEOCH, J. A.: The Influence of the Degree of Learning upon Retroactive Inhibition. *Psychological Bulletin*, Vol. 25, No. 3, pp. 160-161, March, 1928.

The influence of degree of learning upon retroaction was studied with five widely varying degrees of learning, viz. six, eleven, sixteen, twenty-one, and twenty-six repetitions. With a large number of presentations, overlearning was very great. The relative amount of retroactive inhibition measured in terms of both recall and relearning scores, varies inversely as the number of repetitions. Even lists learned to twenty-six repetitions suffered a considerable disintegration, when interpolated lists were used.

MEEKS, LOIS H.: Learning and Retention in Young Children. Teachers College, Columbia University Contributions to Education, No. 164, pp. 86, 1925.

A distribution of practice, making the more subtle analysis a gradual development, is more efficient for young children than concentrated extended practice.

MEUMANN, E.: The Psychology of Learning. An experimental investigation of the Economy and Technique of Memory. Translated by J. W. Baird, D. Appleton and Company, pp. 393, 1913.

We remember only what we have apprehended attentively and with the intention of remembering it. It is most important that the child should receive a hint as to the difference between aimless ineffective repetition and that sort of repetition in which attention and memory receive the full benefit of every single reading. This study contains a splendid bibliography.

MULHALL, E. F.: Experimental Studies in Recall and Recognition. *American Journal of Psychology*, Vol. 26, pp. 217-228, 1915.

"The value of a single presentation is greater in recognition than in recall, and we may add that the value of repetitions is greater for material with meaning."

MÜLLER, G. E., and SCHUMANN, F. S.: Experimentelle Beiträge zur Untersuchung des Gedächtnisses. *Zeit. f. Psycho. d. Sinnesorgane*, Vol. 6, pp. 81-257, 1894.

Their study verifies the work of Ebbinghaus, however they found less forgetting than their predecessors. They found that those who learned nonsense syllables in the shortest time also relearned them in the shortest time.

MYERS, G. C.: A Comparative Study of Recognition and Recall. *Psychological Review*, Vol. 21, pp. 442-456, 1914.

Tested three hundred fifty children with ten words. Ten months later the recall and recognition tests were given. Twenty words were given including the ten previously given. Recognition efficiency is about two and one-half times that of recall.

MYERS, G. C.: Confusion in Recall. *Journal of Educational Psychology*, Vol. 8, pp. 166-175, 1917.

Delayed recall often causes facts to be given in a very confused way, particularly in history and formal logic. In learning, the wrong associates, as far as possible, should be prevented and every care should be taken to fix the right associates beyond doubt.

MYERS, G. C.: Recall in Relation to Retention. *Journal of Educational Psychology*, Vol. 5, pp. 119-130, 1914.

Three hundred thirty-two boys and girls of normal school, academy, and seventh and eighth grades were used. Ten words were dictated with regular tempo. The pupils were asked to write as many words as they could recall after a few minutes, an hour, and three weeks. There was a decided gain in the final recall, as a result of intervening recalls, without re-presentation of the stimuli.

MYERS, G. C.: Delayed Recall in History. *Journal of Educational Psychology*, Vol. 8, pp. 275-288, 1917.

An attempt to make a preliminary measure of the recall of one-time familiar facts in history after the lapse of a year or more. He concludes that wrong answers deserve more careful study; in many cases they give more information than the study of correct answers.

MYERS, G. C., and C. E.: Reconstructive Recall. *American Journal of Psychology*, Vol. 27, pp. 493-506, 1916.

Six subjects were asked to introspect and note the conditions under which recalls were made of selections that they once had been able to quote, but which in the meantime they had entirely forgotten.

NOONAN, M. E.: The Influence of the Summer Vacation on the Abilities of Fifth and Sixth Grade Children. *Teachers College, Columbia University Contributions to Education*, No. 204, pp. 103, 1926.

Conclusion: Current opinion has greatly exaggerated the amount of deterioration that takes place. This interval does not cause any significant change in the ability to compute, read, spell, solve problems, and draw.

NORSWORTHY, N.: The Psychology of Mentally Deficient Children, p. 238, 1906.

One hundred fifty-nine normal and sixty-eight feeble-minded children were used. 5% of the feeble-minded did as well with the related words and 6% with the unrelated words as did 50% of the normals.

NORSWORTHY, N.: Acquisition as Related to Retention. *Journal of Educational Psychology*, Vol. 3, pp. 214-218, 1912.

Material used was a German-English vocabulary of 1200 words with eighty-three students in Educational Psychology. The method differs from those hitherto reported, in that the time remained constant and the amount learned varied.

PIERON, H.: Recherches experimentales sur les phenomenes de memoire. *L'Année psychol.*, Vol. 19, pp. 91-93.

His results agree with those of Ebbinghaus.

PYLE, WILLIAM H.: Retention as Related to Repetition. *Journal of Educational Psychology*, Vol. 2, pp. 311-321.

Pyle was particularly interested in determining the relation between memory and quickness of learning. He used twelve seniors and graduate students of the University of Missouri. The material was presented to each individual and he was required to give it orally after each presentation. The most rapid learner showed the highest percentage of retention and was also the most accurate.

RADOSSAWLJEWITSCH, P. R.: Das Fortschreiten des Vergessens mit der Zeit. Göttingen, Druck der Dieterich'schen Univ.—Buchdruckerei, pp. 197, 1905.

The initial rate of forgetting is not nearly so fast as Ebbinghaus had found, although it proceeds rather quickly at first, but thereafter considerably more slowly.

REED, H. B.: Associative Aids: Their Relation to Learning: Their Relation to the Theory of Thought and to Methodology in Psychology. *Psychological Review*, Vol. 25, pp. 128-155, 1918.

The absence of associative aids produces slow learning, and quick forgetting. High retention may be due to strong imprinting as well

as to the presence of an association. The rate of relearning varies directly as the rate of learning.

SEIBERT, LOUISE C.: An Experiment in Learning French Vocabulary. *Journal of Educational Psychology*, Vol. 18, pp. 294-309, 1927.

What is the relative efficiency of the following methods? (a) learning silently, (b) learning aloud, (c) learning aloud with immediate recall. "Its findings are that studying aloud is the best method."

SHELLOW, SADIE: Individual Differences in Incidental Memory. *Archives of Psychology*, No. 64, pp. 77, 1923.

This work contains a summary of her own conclusions and those of other investigators on the subject of recognition and recall.

SMITH, M., and McDUGALL, W.: Some Experiments in Learning and Retention. *British Journal of Psychology*, Vol. 10, pp. 199-210, 1920.

Five tests were given: (1) The Blot test, (2) Picture test, (3) Prose test, (4) Learning of nonsense syllables, (5) The typewriter test. Four women students were used. Conclusion: The retention is more effective when the learning is done actively and not passively.

STRATTON, G. M.: Retroactive Hypermnnesia and Other Emotional Effects on Memory. *Psychological Review*, Vol. 26, pp. 474-486, 1919.

Had students of the University of California write their recollections of the earthquake of 1906 in San Francisco. Emotions seem to strengthen the connections by which former experiences may be called into life.

STRONG, M. H., and E. K.: The Nature of Recognition Memory and of Localization of Recognitions. *American Journal of Psychology*, Vol. 27, pp. 341-362, 1916.

Recognition is due first of all to the fact that when an object is seen again, it arouses the same associations that it did when it was first seen. The ability to localize fades out more rapidly than the ability to recognize.

STRONG, E. K., JR.: The Effect of Time Interval upon Recognition Memory. *Psychological Review*, Vol. 20, pp. 339-372, 1913.

There was an exposure list of twenty common English words which the subject read aloud and then solved a short problem in mental arithmetic. A second list of forty words was given him from which he checked the words which he remembered. Thirteen time intervals were used ranging from fifteen seconds to seven days, for two of the five subjects there was an interval of forty-two days. He obtained a relationship between recognition memory and the length of time between exposure and identification practically identical to that found by Ebbinghaus.

STRONG, E. K.: The Factors Affecting a Permanent Impression Developed through Repetition. *Journal of Experimental Psychology*, Vol. 1, pp. 319-338, 1916.

Advertisements were shown and retention was tested after a day, a week, a month, and sixteen weeks. With an increase of the number of advertisements comes a decrease in the effectiveness of the permanent impression. After the first two days this impression seems to die out gradually.

STRONG, E. K.: Two Factors which Influence Economical Learning. *Journal of Phil., Psy., and Sci. Methods*, Vol. 11, pp. 124-131, 1914.

The two factors are (1) How long shall one work at any one period? (2) How long an interval of time shall intervene between successive periods? Conclusions: (1) Successive repetitions of a day's length will give maximum results in advertising. (2) The more impressions made at one time, the less is the permanent retention of any of them.

Supplemental Report. S. R. 711, Part 2-56 Cong. School Examinations. 1st sessions, pp. 15, 1900.

1188 first year high school pupils of Washington, D. C., were examined on common school branches which had not been taught for nine months. 40% of the grammar school content could not be recalled.

SWIFT, E. J.: Memory of a Complex Skilful Act. *American Journal of Psychology*, Vol. 16, pp. 131-133, 1905.

After six hundred forty-two days Subject A had forgotten little or nothing of the ability to keep two balls going with the right and two with the left. Subject E allowed six hundred sixteen days lapse for the right hand and six hundred twelve days for the left with the same results.

SWIFT, E. J.: Relearning a Skilful Act. *Psychological Bulletin*, Vol. 7, pp. 17-19, 1910.

Six years and seventeen days after the experiment noted above, Swift repeated the investigation. He found that eleven days were required to regain the skill which in the initial learning had taken forty-two days of practice.

TAIT, WILLIAM D.: The Effect of Psycho-Physical Attitude on Memory. *Journal of Abnormal Psychology*, Vol. 8, pp. 10-37, 1913.

The cause of any existing attitude may be said to be in the response to some previous impression or impressions with a high emotional and feeling tone which were not allowed an adequate expression in the normal way, at the time.

THORNDIKE, E. L.: The Relation between Memory for Words and Memory for Numbers and the Relation between Memory over Short and Memory over Long Intervals. *American Journal of Psychology*, Vol. 21, pp. 487-488, 1901.

The correlation between brief and long retention in the case of unconnected material was found to be about .9. The ability tested was that of remembering sixty words given in five tests of twelve words each, twenty-four hours later.

THORNDIKE, E. L.: Memory for Paired Associates. *Psychological Review*, Vol. 15, pp. 122-138, 1908.

Twenty-two adults, seniors or graduate students, were given one hundred twenty sets of ten each German words with their paired English equivalents. After a student had spent hours on these words and was sure he knew them, then he was tested. Conclusion: "It would not be far amiss to say, that the central tendency would be a loss of one-twentieth in an hour, one-tenth in three days, and between four and five tenths in forty days."

THORNDIKE, E. L.: Permanence of Ability to Recite a Series of Nonsense Syllables. *Educational Psychology*, Vol. II, Teachers College, Columbia University, New York, 1913.

A survey of the work done by Ebbinghaus, Radossawljewitsch, Bean and others. Conclusion: "Indeed the only facts that results display are those of the variability of deterioration by disuse and the lack of knowledge about it."

THORNDIKE, E. L.: The Permanence of School Learning. *School and Society*, Vol. 15, pp. 625-27, 1922.

One hundred eighty-nine graduates in their first year law school, who had had no algebra for four or five years, were given five examples in algebra to do in ten minutes. The scores made in this test were compared with the scores made by the top half of the entering college freshman class in the algebra part of the Thorndike Intelligence Examination for high school graduates. Conclusion: "So far as I can ascertain, nobody has ever measured the extent of the actual forgetting."

Trow, W. C.: Recall vs. Repetition in the Learning of Rote and Meaningful Material. *American Journal of Psychology*, Vol. 40, No. 1, pp. 112-116, 1928.

Ten graduate students in educational psychology worked with three kinds of materials, (1) series of digits, (2) lists of monosyllabic concrete and abstract nouns, and (3) a descriptive paragraph. The methods of recall used for practice were as follows: I. Control group—at each practice period they wrote down as much as they could remember without hearing the material. II. Recall group—at each practice period they wrote down as much as they could remember from the previous time and then listened to the material again. III. Re-presentation group listened and then wrote. IV. Recall representation group wrote, listened, and wrote again. V. Massed group listened then wrote. Some of the conclusions were: Method III is consistently ineffective. The Massed method is as good as any for meaningful material and better than all others for rote material for one day's retention, but quite useless for retention over much longer periods.

TSAI, CHIAS: A Comparative Study of Retention Curves for Motor Habits. *Comparative Psychology Monographs*, Williams and Wilkins Co., Baltimore, Maryland, pp. 29, October, 1924.

Retention of the stylus maze habit was tried with seventy-four men and women in the elementary class in psychology. A high degree of variability was noted.

TSAI, LUH-SENG: The Relation of Retention to the Distribution of Rerearning. *Journal of Experimental Psychology*, Vol. 10, pp. 30-39, 1927. Experiment I.

Ten paired associates, an English noun and its corresponding trade name were used with one hundred twenty-one Chinese and one Korean college student. His returns exhibited negative acceleration, with the most rapid forgetting in the first two days. Experiment II. Investigation of the distribution of learning and rerearning was tried out with seventy-two students in a class of elementary psychology. These students were divided into three groups which learned and rerelearned the material, in nine trials, distributed over eleven days. Retention was tested after three and five days.

WAGNER, A. E.: An Experiment to Determine the Number of Repetitions Necessary to Memorize and Retain with Maximum Certainty. A Thesis, University of Pennsylvania, pp. 37, 1910.

A special set of geography questions, the answers of which seemed unknown to the pupils of the fifth, sixth, seventh, and eighth grades, was prepared. The investigator presented all the questions for a particular class at one sitting. The answer was written on the blackboard. At each repetition, the pupils wrote the answers—one to three words—on slips of paper. They marked and corrected their errors. The fifth graders knew their answers at end of fourth repetition, sixth graders, at end of fifth, seventh graders, at end of fourth, and eighth graders, at end of third.

WATTS, S.: The Qualitative Investigation of Higher Mental Processes. *British Journal of Psychology*, Vol. 6, pp. 109-133, 1913.

Thirty-four children between the ages of eleven and thirteen years were tested out for immediate memory and retention, after two days of nonsense syllables. The coefficient of correlation between the estimated intelligence and the immediate recall test was .59 and between intelligence and the delayed recall test was .74. Watts states that the coefficient of correlation between intelligence and retention after longer intervals of time would have been much higher.

WHIPPLE, G. M.: Notes on the Recall of Nonsense Syllables. *Journal of Educational Psychology*, Vol. 1, pp. 404-405, 1910.

After nine months, a Mr. C. W. Hagen sent the whole list to Whipple with just two minor errors.

WOHLGEMUTH, A.: The Influence of Feeling on Memory. *British Journal of Psychology*, Vol. 13, pp. 405-416.

Conclusion: Memory is not influenced at all by the feeling of pleasure or displeasure.

WOOD, E. L.: An Experimental Analysis of the Process of Recognizing. *American Journal of Psychology*, Vol. 26, pp. 313-387, 1915.

Two objections to the facilitation theory: (1) It helps us not at all on the side of consciousness. (2) The most facile processes—those which accompany the daily habitual experiences to which our reactions are almost automatized—do not bring with them the peculiar experience of familiarity.

WOODY, CLIFFORD: Scores made by Seniors on the Hotz Algebra Scales compared with Scores made by High School Students Taking Algebra. *School and Society*, Vol. 16, pp. 303-306, September 9, 1922.

Conclusion: The seniors have retained a relatively large amount of the formal aspects of algebra.

WOODYARD, ELLA: The Effect of Time upon Variability. *Teachers College, Columbia University Contributions to Education*, No. 216, 1926.

From the results of a battery of fifteen tests, the conclusion is that the time factor is of small moment in causing an individual to vary in the mental work he produces under conditions found in the administration of such standard tests as are common in intelligence and educational testing.

WORCESTER, DEAN A.: The Permanence of Learning in High School Subjects. *Journal of Educational Psychology*, pp. 343, May 1928.

"How well is algebra remembered? A serious problem is to provide practice sufficient for permanent retention for content given late in the year, or shall we just 'leave off the last car'?"

WORCESTER, DEAN A.: Retention after Long Periods. *Journal of Educational Psychology*, Vol. 14, pp. 113-114, 1923.

The writer, himself, learned in 1915, twenty-one one hundred word selections from the works of Arnold and Huxley, by using the auditory method for twelve of these, and the visual for the remaining nine. Five years later he found that in the relearning of these selections there was a saving of 44.45% by the auditory method and 43.09% by the visual.

B-VITA.

Annabel Lee White was born on April 25, 1884, in Baltimore, Maryland. She received her elementary and secondary education in the public schools of Baltimore. She was graduated from the Western High School in 1902 and from the Teachers' Training School in 1903.

She taught in the elementary grades from 1903 to 1907, when she became a teacher of mathematics in one of the preparatory schools of the city. From there, in 1909, she went to the Western High School to teach mathematics. In 1920 she became head of the department of mathematics at that school. In January 1927, she became vice-principal and dean of girls at the Forest Park High School.

While she has studied at Cornell and Columbia during the summer months, practically all of her undergraduate and graduate work has been done at the Johns Hopkins University. In 1923, she received the degree of Bachelor of Science; in 1926, the degree of Master of Arts, and in 1930, the degree of Doctor of Philosophy.

She feels she owes a debt of gratitude to the late Dr. E. F. Buchner for making it possible for her to do university work while holding her position as a teacher.

